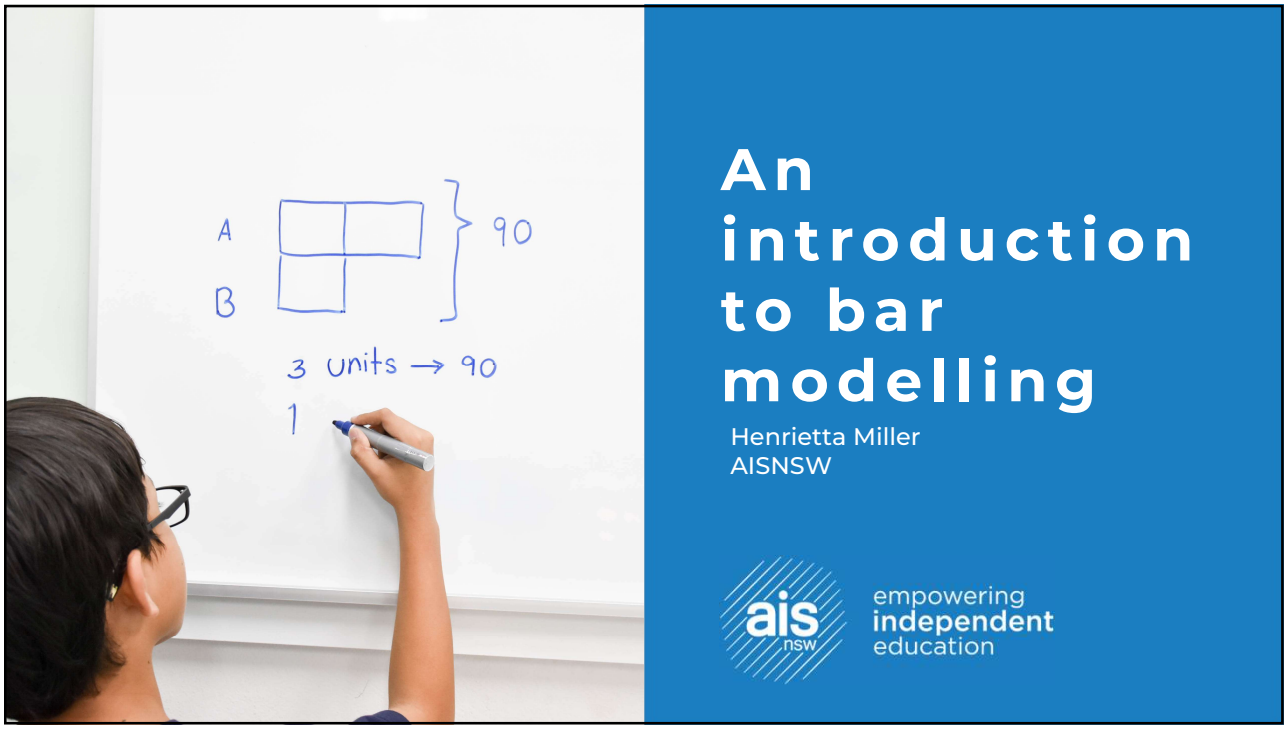


An introduction to bar modelling

Henrietta Miller
AISNSW

ais nsw empowering independent education

1

What will be in this presentation?

- The origins of bar modeling
- An overview of the bar modelling process
- Top tips for teachers in how and when to use bar models
- Resources to support teachers.

2

Bar Modelling

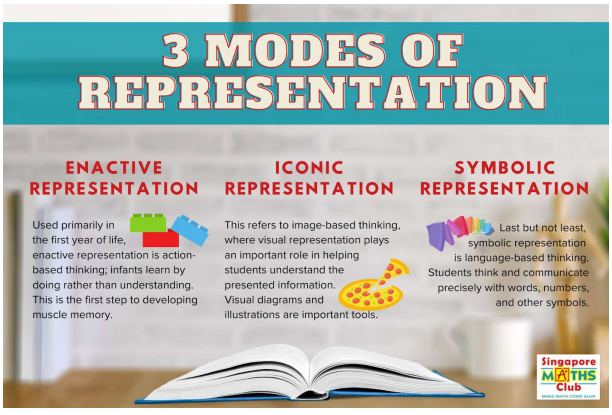
The theory underpinning bar model is known as:

Concrete, Pictorial or Visual, Abstract (CPA or CVA)

3

‘**The Singapore Maths Model**’ another name for the **bar model** method, is based heavily on the work of Bruner, Dienes and Bishop.

Third Space learning



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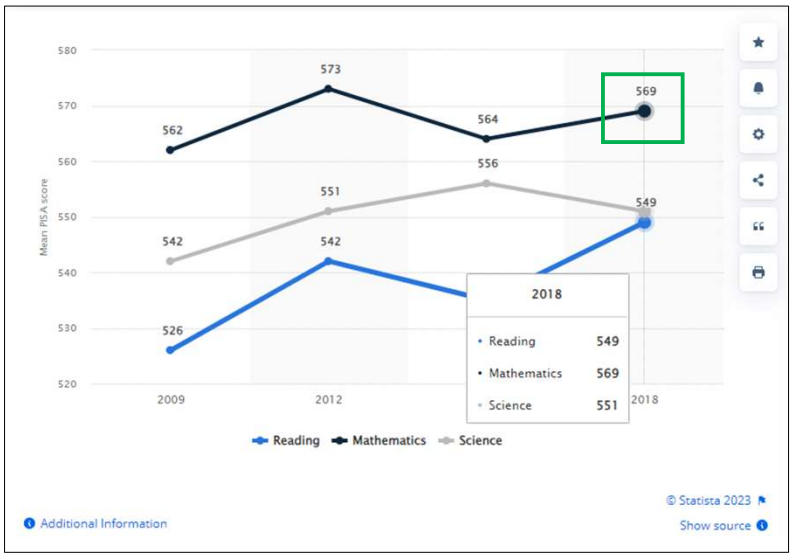
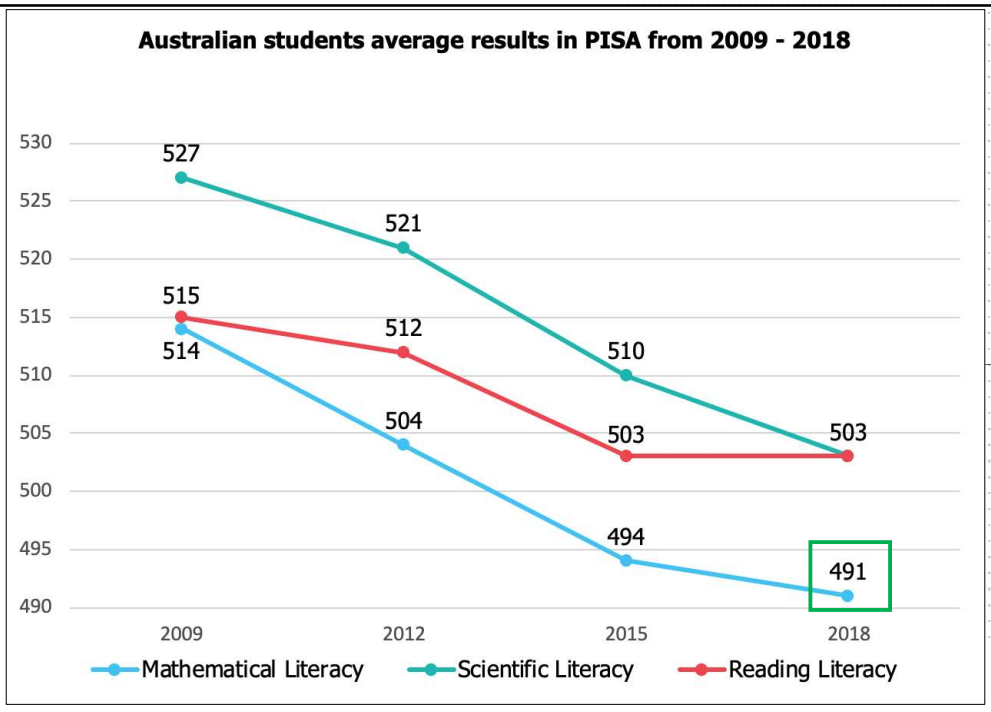


Mean Programme for International Student Assessment (PISA) score of students in Singapore from 2009 to 2018, by subject

5



6

Concrete – Visual – Abstract (CVA) approach

A system of learning that uses physical and visual aids to ***build*** a child's understanding of abstract topics.

It provides teachers with opportunities to:

- model mathematical thinking using manipulatives
- build a child's overall ***sense*** of number which allows for transfer ***across*** a range of mathematical skills

Concrete

Visual

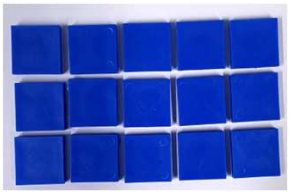
Abstract

7

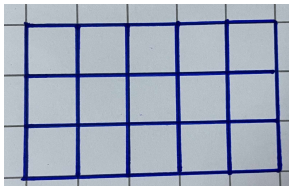
Concrete-Visual-Abstract sequence

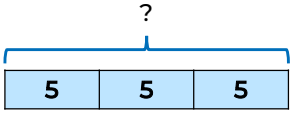
All students, regardless of ability, benefit from the use of practical resources in ensuring understanding goes beyond the learning of a procedure.

Concrete representation
Use square counters to represent 3 x 5, showing rows and columns.



Visual representation
Draw 3 x 5 using grid paper or using a bar model





Abstract representation
Represent the problem in words and symbols

There were five children at a party. They each ate three cupcakes. How many cupcakes did they eat altogether?

Represent the problem using numbers and symbols

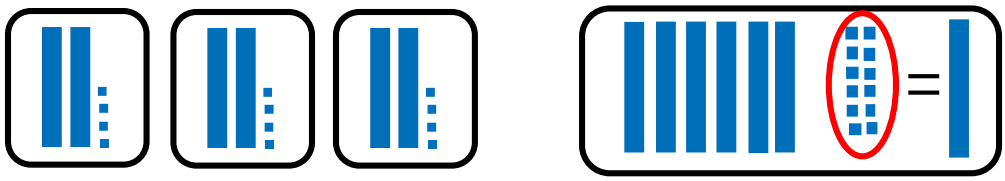
3 x 5 = 15

8

Concrete-Visual-Abstract sequence

If Joe has 24 stickers and Sally has three times as many stickers as Joe, how many stickers do they have altogether?

Use MAB blocks to model and demonstrate that there are three groups with 24 in each group



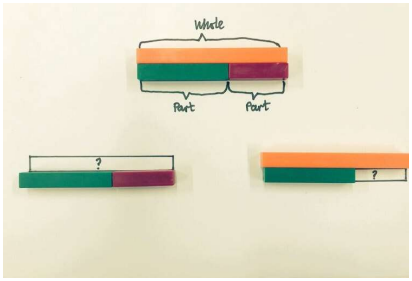
Then use a **bar model** to represent the three groups with 24 in each group

Joe	24		
Sally			

9

Concrete-Visual-Abstract sequence

- These three phases are designed to overlap.
- It's not an entirely linear process where students work only with concrete materials, then move to visual representations, and finally use only abstract methods and symbols.
- Students need to make connections between the abstract and the concrete or representational, otherwise the symbols remain abstract.



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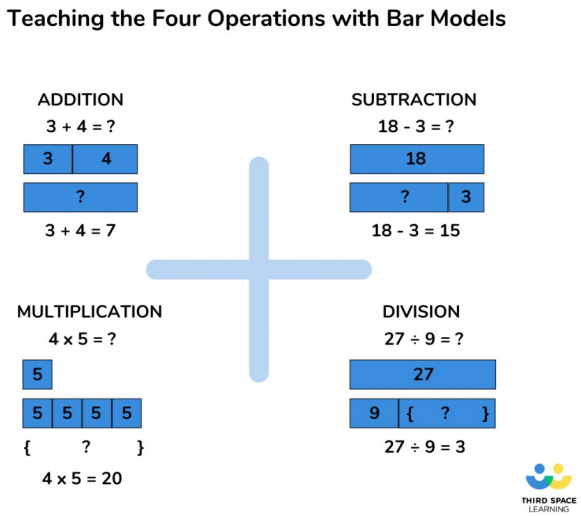
Bar Modelling

is a drawing used as part of the **Concrete, Visual Abstract** approach to teaching.

Bar models are a visual representation of a problem where bars are used to represent the **known** and **unknown** quantities.



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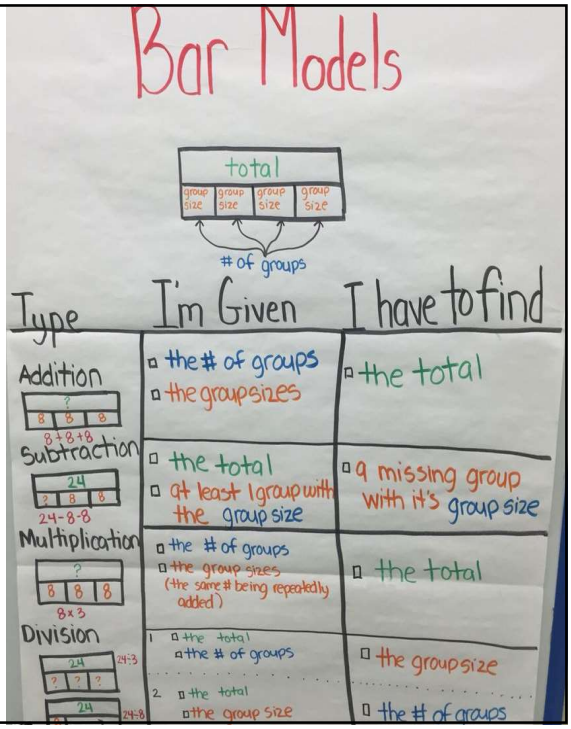
Bar Modelling

- Bar models **will not 'do'** the maths, they allow students to understand what maths needs to be done.



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Bar modelling

Bar modelling does not require students to have any resources except for pencil and paper.

For example, bar models can be used in the classroom and in tests when other resources are not allowed.

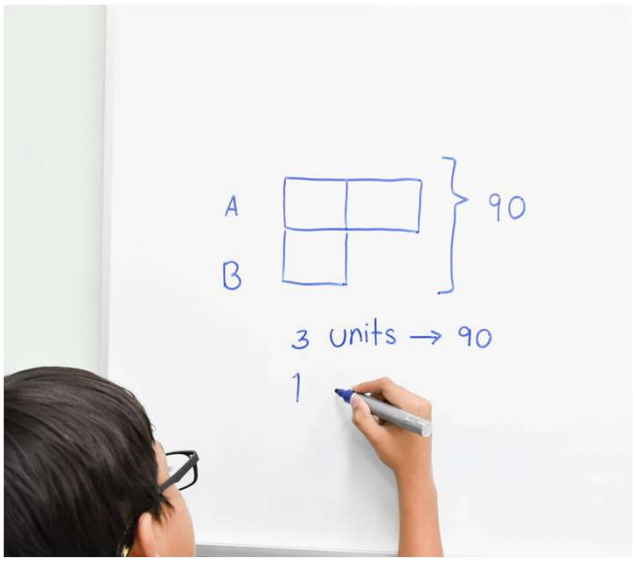
Addition	Subtraction
7 + 11 = ? 7 | 11 ?	15 – 7 = ? 15 7 | ?
Addition	**Subtraction**
143 + 165 = ? 143 | 165 ?	289 – 187 = ? 289 187 | ?

13

Bar modelling

Is a form of mathematical drawing using rectangles.

These rectangles are a representation not a precise drawing.



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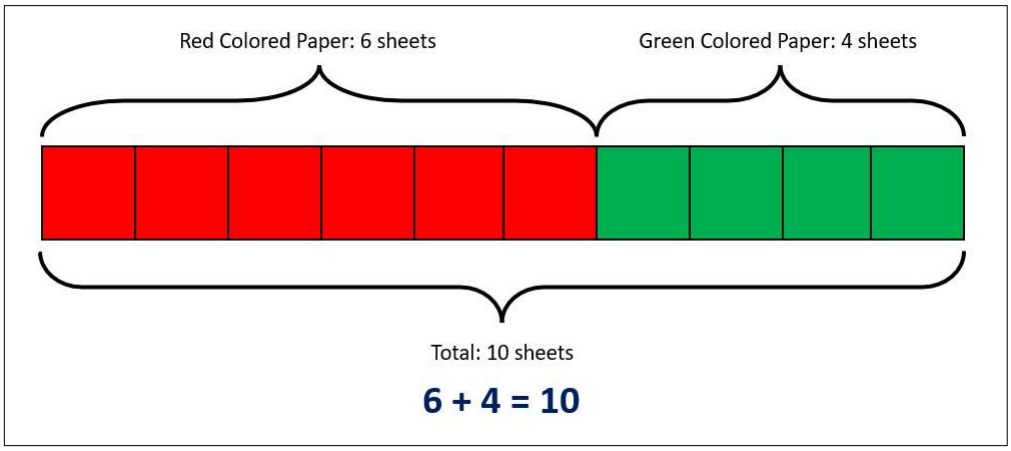
Bar models are also known as tape diagrams (in Japan) and strip diagrams (in the US).



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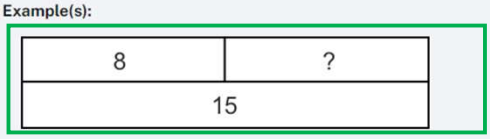
Stage 1B

Model how addition and subtraction are inverse operations using concrete materials, drawings and diagram.

Example(s):



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Example(s):

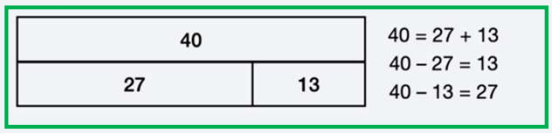

Image long description: A bar model is made up of three rectangles, representing the part-part-whole relationship between 27, 13 and 40. One on top is labelled 40, and two rectangles below are labelled 27 and 13, and are collectively the same length as the first rectangle. The equations '$40 = 27 + 13$', '$40 - 27 = 13$' and '$40 - 13 = 27$' are to the right.

Stage 2A examples

Example(s):

If 64 – 17 = 47, what is 64 – 47?

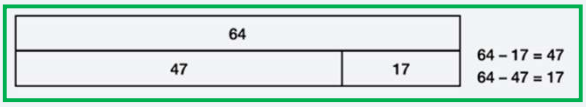

Image long description: A bar model is made up of three rectangles, representing the part-part-whole relationship between 47, 17 and 64. One on top is labelled 64, and two rectangles below are labelled 47 and 17, and are collectively the same length as the first rectangle. The equations '$64 - 17 = 47$' and '$64 - 47 = 17$' are to the right.

17

In Stage 2, fractions are represented by **partitioning lengths**. This enables students to measure with partitioned unit fractions. Mapping these lengths to number lines and then aligning the zero point on number lines provides a basis for comparison of fractions as abstract numbers.

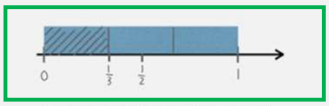

Figure 4: Comparing one-third to one half

Stage 2A Partitioned fractions teaching advice

Example(s):

Compare unit fractions using aligned diagrams and number lines to determine which is larger $\frac{1}{3}$ or $\frac{3}{8}$?

Image long description: The first fraction strip is split into eighths shaded up to 3 eighths overlaid on a number line with eighths marked. The second fraction strip is split into thirds shaded up to one third overlaid on a number line marked with thirds. The 2 fraction strips show that 3 eighths is larger than one third.

Stage 2B Partitioned fractions example

18

Example(s):

A recipe requires 3 cups of flour for 10 people. How many cups of flour are required for 40 people?

Image long description: A rectangle divided into quarters with the first quarter stating 3 cups for 10 people, with an unknown number of cups for 40 people at the end of the rectangle.

Example(s):

Find

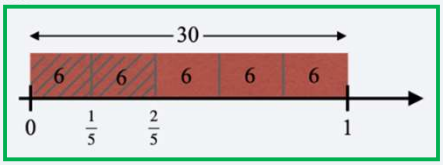

Image long description: A fraction strip divided into fifths overlaid on a number line from 0 to 1 split into fifths, each section of the fraction strip is labelled as 6, with 2 fifths shaded, and the number 30 overhanging the length of the number line.

Stage 3A Multiplicative relations example

Stage 3B Representing quantity fractions example

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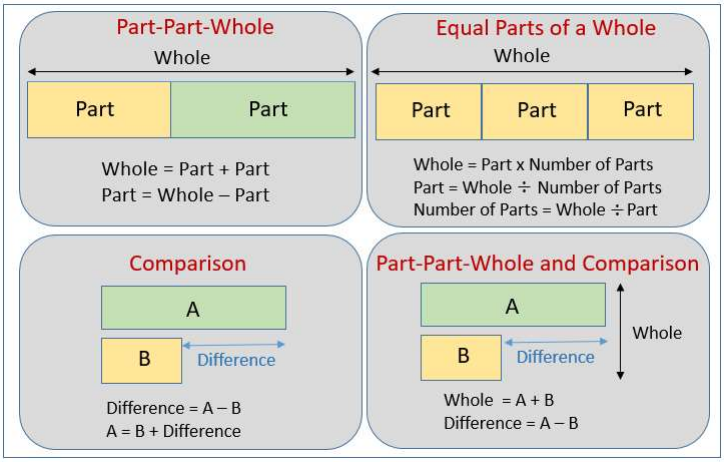
Bar Modelling



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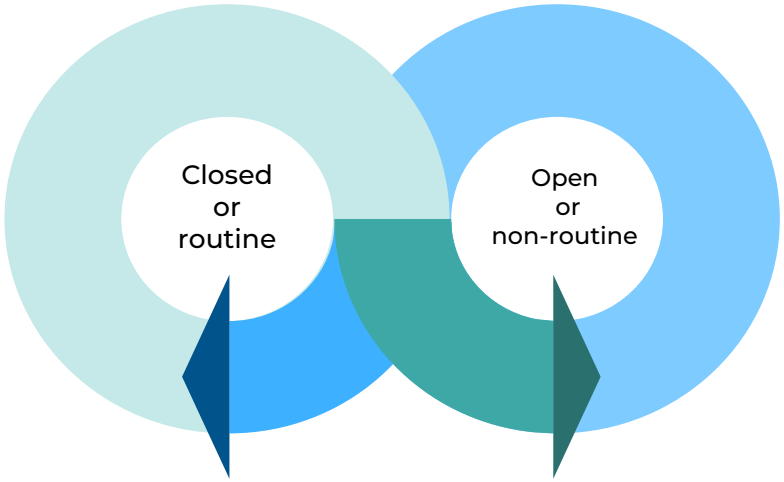
Types of problems



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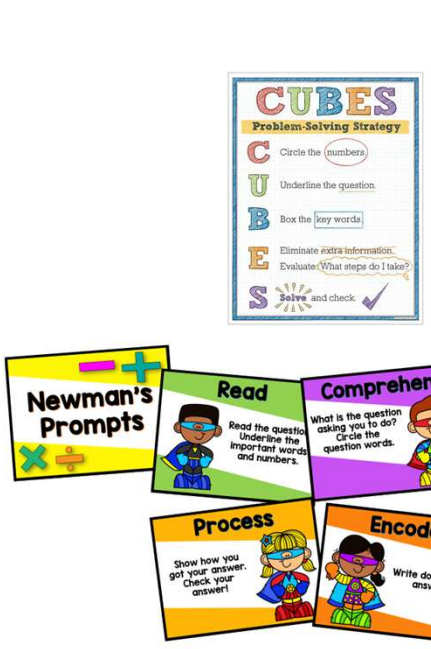
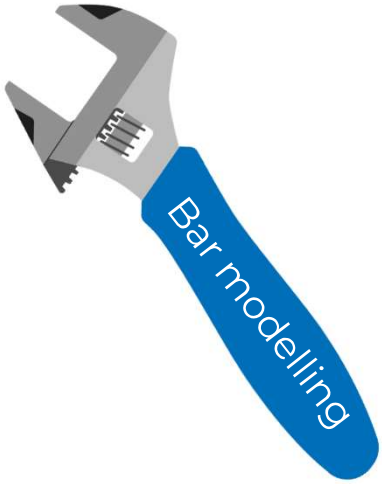


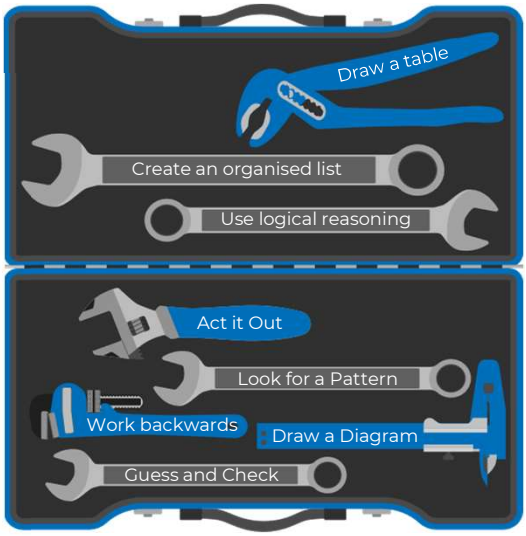
The **RUCSAC** Method for solving maths word problems

R	Read the question carefully	Find the important information – underline it!
U	Understand the question	What do you have to find out? Draw a 'picture' of the question, if it helps.
C	Choose the correct method of calculation	+ − × ÷ What method is best for you to use?
S	Solve the problem	Show every step and keep your working out neat.
A	Answer the question	Read the question again – have you answered it? Make the answer clear.
C	Check your answer	Does it make sense? Find a way to check – estimate or use the inverse.

22

Students need a toolbox of strategies to solve problems





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Types of Bar Modelling

Part/Whole

?	
17	83

Bar model representing the addition equation 17 + 83 =

160	
?	40

Bar model representing the subtraction equation 160 – 40 =

36			
?	?	?	?

Bar model representing the division equation 36 ÷ 4 = or the multiplication equation 4 x ? = 36

Comparison

11	
7	?

?

Jo has 11 cakes and Sam has 7 cakes. What is the difference? Or how many altogether?

12	
3	?

Sandy has 12 football cards and Umar has 3. How many more does Sandy have than Umar?

24

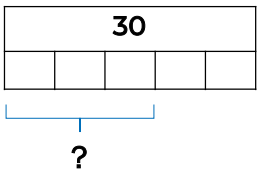
Types of Bar Modelling

Missing number problems

?	
7	9

Bar model to represent the problem 7 = ? – 9

Fraction problems



Bar model representing the equation: What is 3/5 of 30?

Money problems

$3		
55c	60c	?

A boy has $3. He buys some lollies for 55c and some chips for 60c. How much change does he get?

Time problems

From 7:35 to 8:55 = ? minutes		
?	10 min	?

A film starts at 7:35 pm and ends at 8: 55 pm. There is an ice-cream break of 10 minutes, halfway through. How long is the movie?

25

Example 1

My family travelled to Byron Bay for holiday. We drove 476km before lunch and 285km after lunch. How far did we travel altogether?

?	
476	285

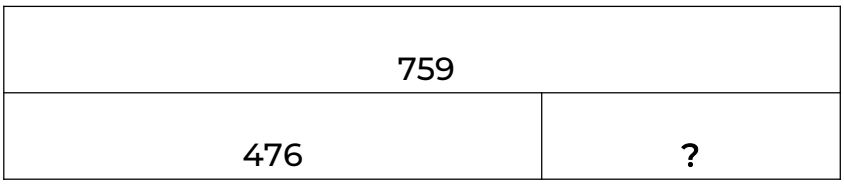
Part/whole or missing number problem

26

Example 2

My family travelled to Byron Bay for holiday. We drove 759km altogether.

If we drove 476 before lunch, how far did we travel after lunch?

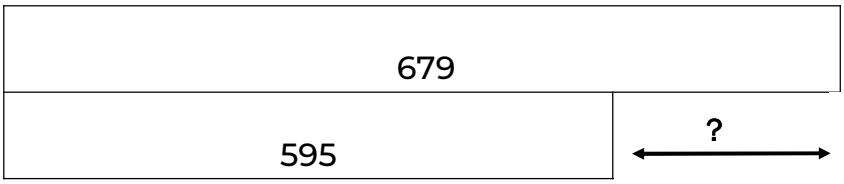


Part/whole or missing number problem

27

Example 3

My family and some friends travelled in two cars on holiday. We drove 595km in one day and they travelled 679km in one day? How much further did they travel than us?



Comparison problem

28

Example 4

Josie had 25 cupcakes and she needed to share them between five friends

How many cupcakes did each friend receive?

25				
?	?	?	?	?

Part/whole – division

29

Example 5

Five children each had six toy cars. How many cars did they have altogether?

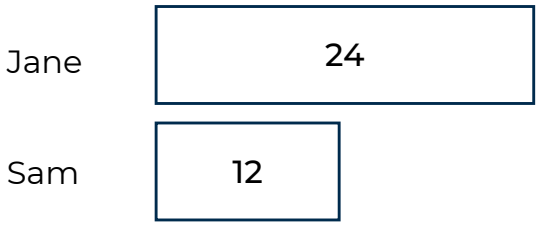
?				
6	6	6	6	6

Part/whole – multiplication

30

Example 6

Jane had 24 apples and Sam had 12 apples, what fraction of the apples are green?

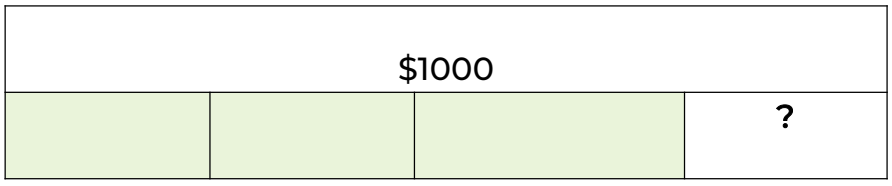


Fraction problem/comparison problem

31

Example 7 – two step problem

I spent $1000 altogether, buying a Nintendo game boy and some games. I spent ¾ of my money on the Nintendo. How much did I spend on games? How much money did I spend on the Nintendo?



Fraction problem

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Example 8 – two step problem

I saved $750. I spent $575 on a Nintendo Game Boy and $125.50 buying some games for it. How much change did I receive?

$750		
$575	$125.50	?

Money problem

33

Example 9 – two step problem

There were 6000 people at the one-day cricket test match seated in 5 sections. 2235 were seated in Section A, 1062 in Section B, 823 in Section C and 227 in section D.
How many people were seated in Section E?

6000				
2235	1062	823	227	?

Missing number problem – addition and subtraction

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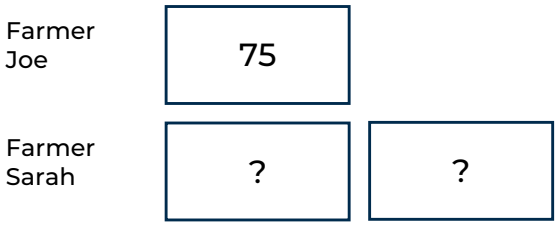
Example 10 – two step problem

Farmer Joe had 75 sheep on his farm. Farmer Sarah had twice as many sheep as farmer Joe. How many sheep did farmer Sarah have? How many sheep did they have altogether?



Comparison problem

35

Your turn

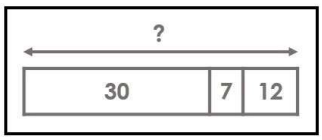
There were 30 grapes in a bowl. Lara ate 7 grapes and Jen ate 12 grapes. **How many grapes left in the bowl?**

Tim had 30 stickers. He gave 7 stickers away and bought 12 more stickers. **How many stickers does he have now?**

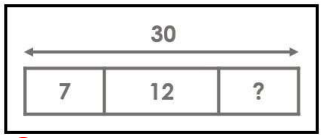
Ben has 30 sweets, Jack has 7 sweets and Jim has 12 sweets. **How many sweets do they have in total?**

Match the image to the problem

A



B



C

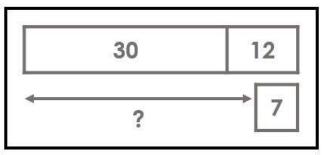


Image thanks to Gareth Metcalf
https://www.iseemaths.com/

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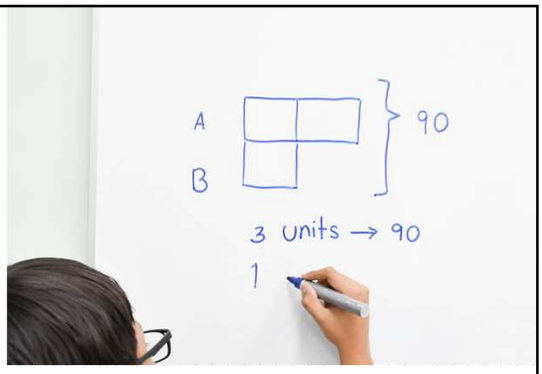
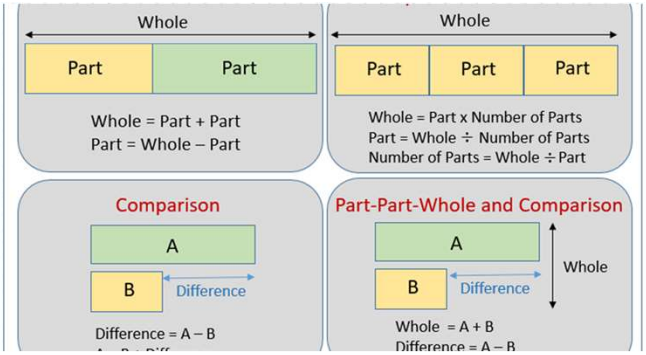
Resources





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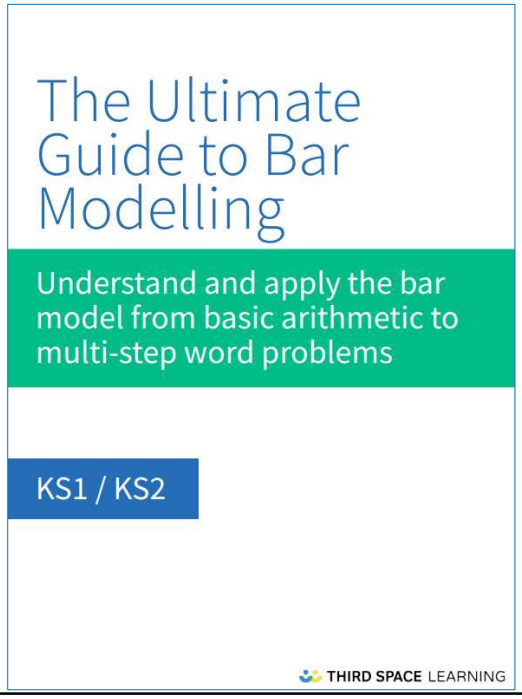
Third Space Learning



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What will my answer look like?

Bar Model Part 1:		**Working out:**
Underlying structure: Whole unknown Size of part(s) unknown Number of parts unknown	**Operation:** + - X ÷	**What I have worked out so far:**
Bar Model Part 2:		**Working out:**
Underlying structure: Whole unknown Size of part(s) unknown Number of parts unknown	**Operation:** + - X ÷	**What I have worked out so far:**

https://thirdspacelearning.com/blog/bar-modelling-techniques-maths-word-problems-sats-multi-step/

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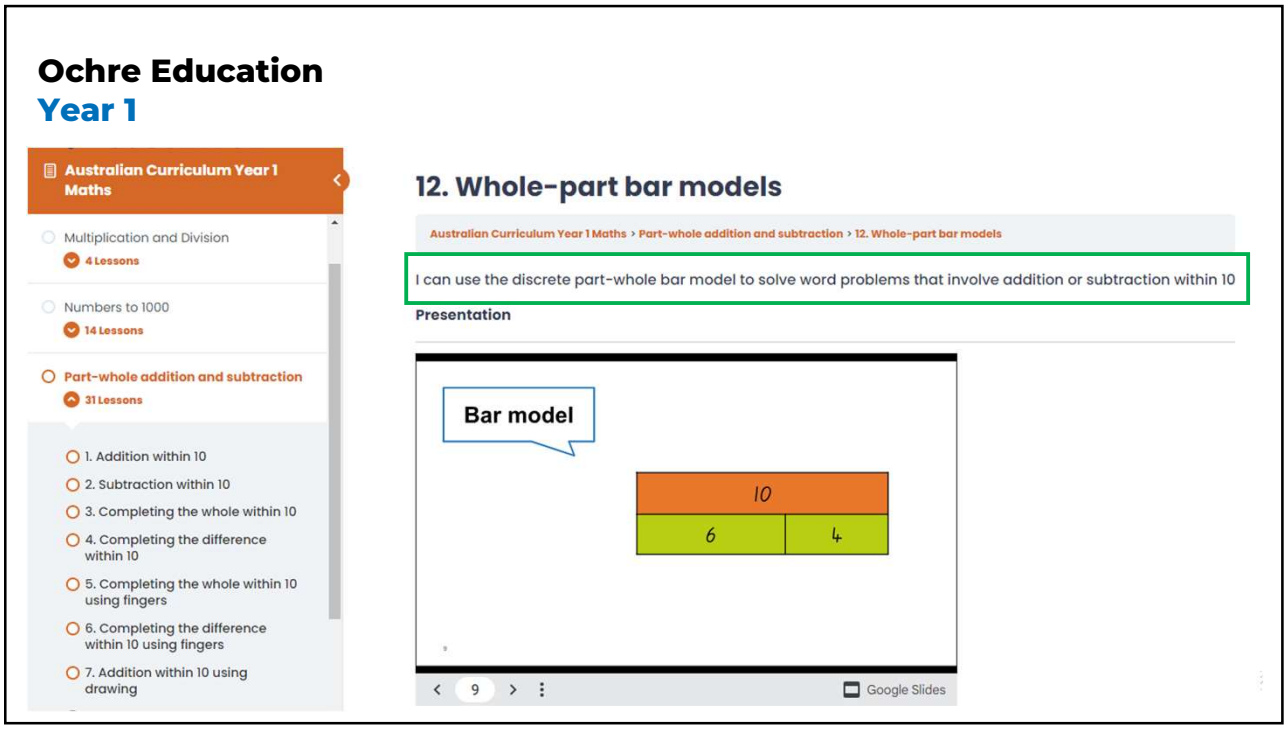

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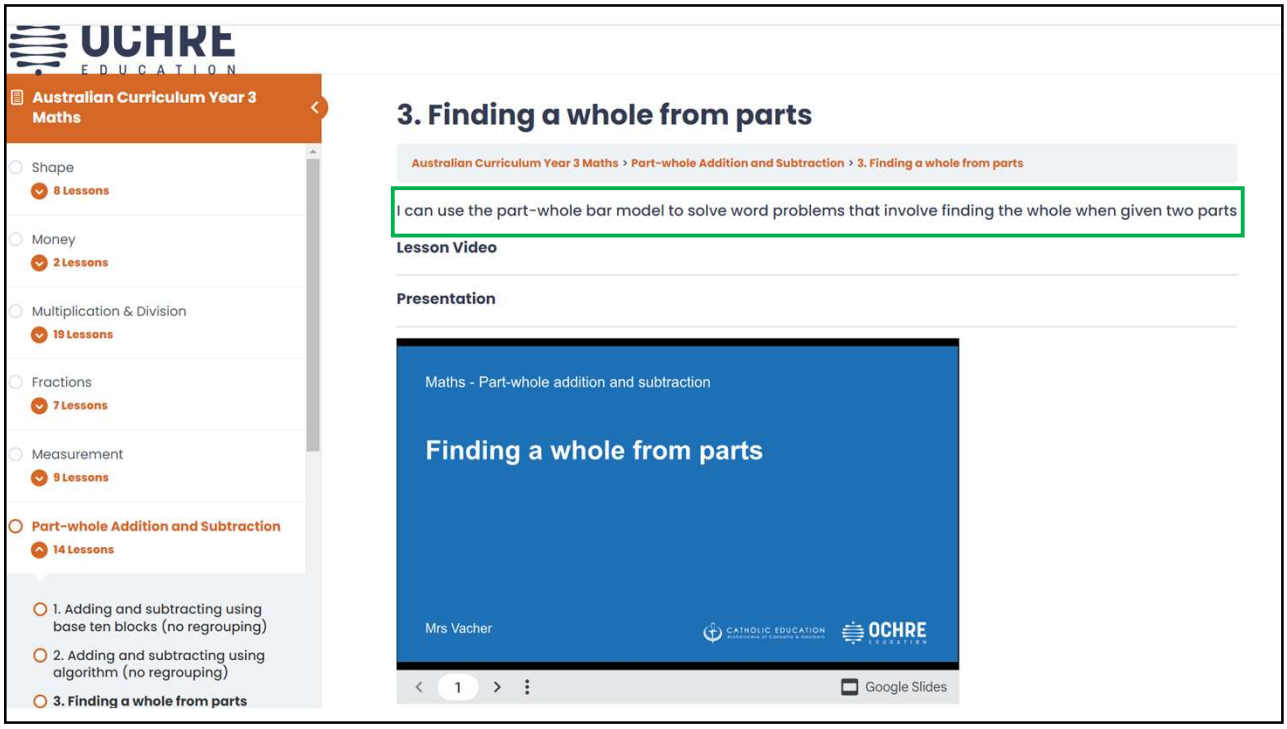

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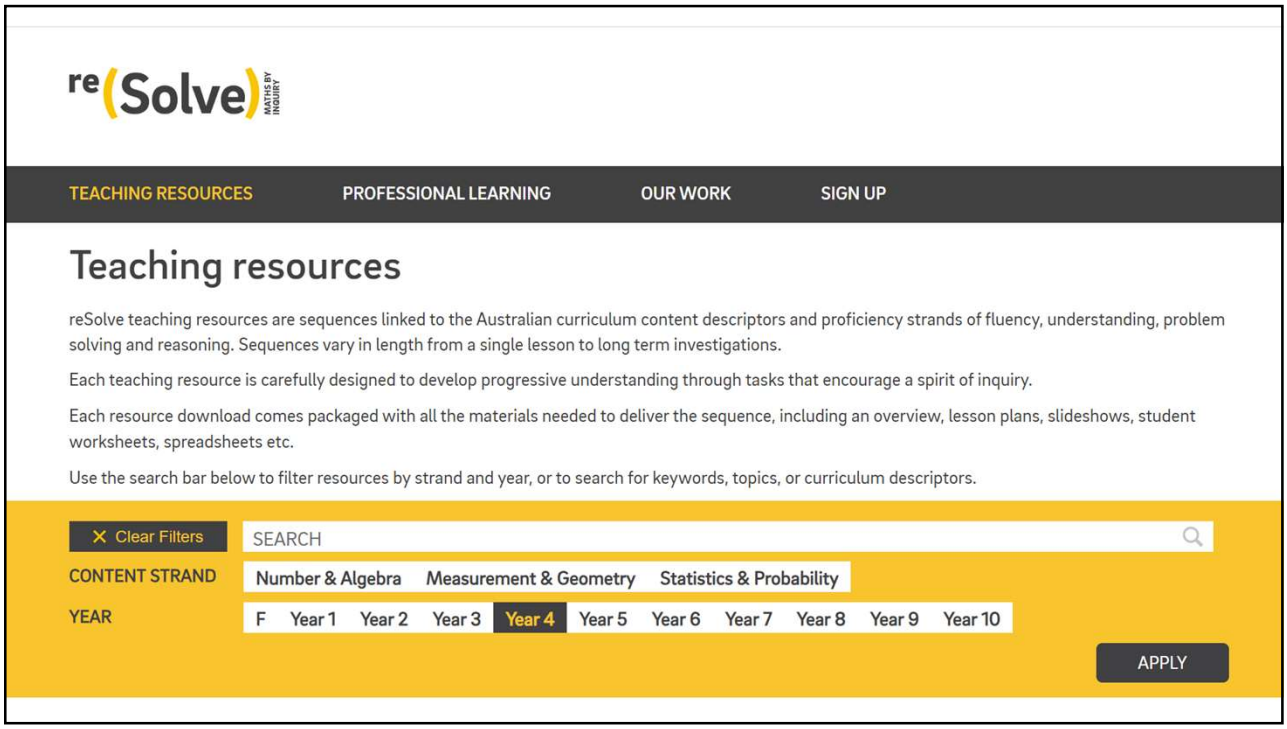
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Year 10
APPLY

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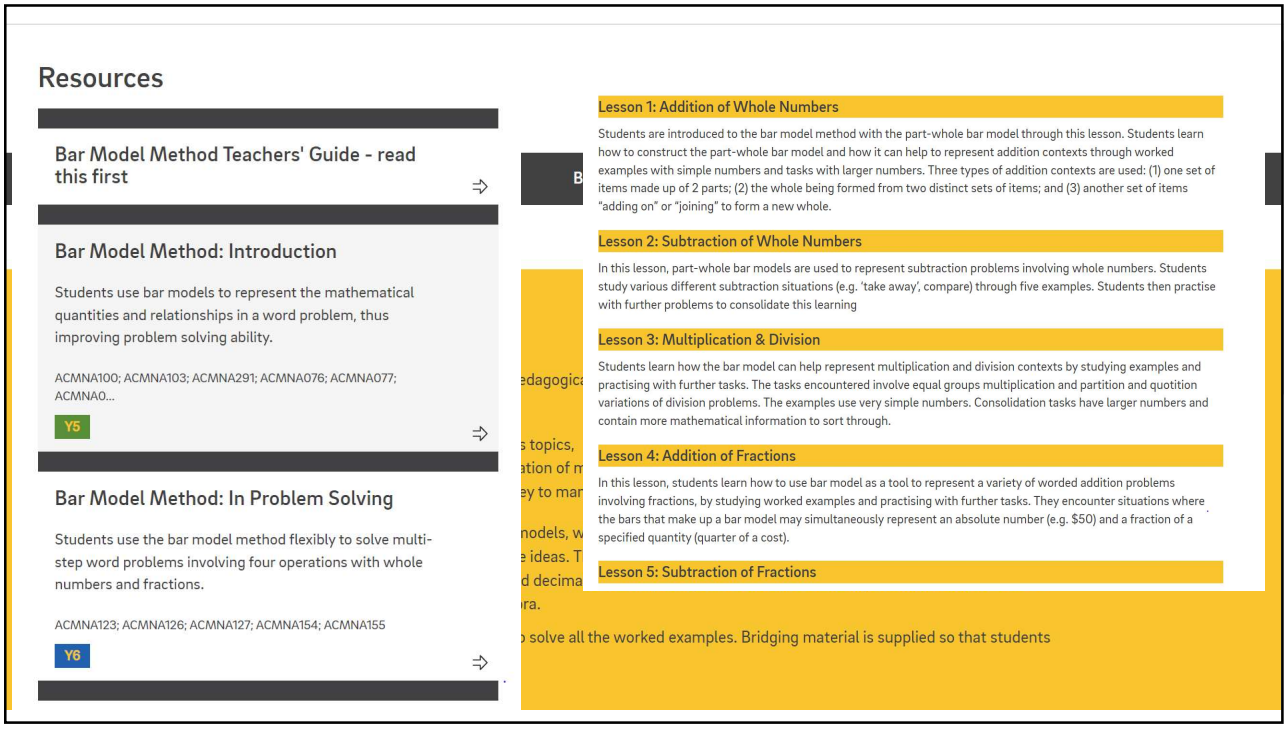
Resources
Bar Model Method Teachers' Guide - read this first
Bar Model Method: Introduction
Students use bar models to represent the mathematical quantities and relationships in a word problem, thus improving problem solving ability.
ACMNA100; ACMNA103; ACMNA291; ACMNA076; ACMNA077; ACMNA0...
Y5
Bar Model Method: In Problem Solving
Students use the bar model method flexibly to solve multi-step word problems involving four operations with whole numbers and fractions.
ACMNA123; ACMNA126; ACMNA127; ACMNA154; ACMNA155
Y6
Lesson 1: Addition of Whole Numbers
Students are introduced to the bar model method with the part-whole bar model through this lesson. Students learn how to construct the part-whole bar model and how it can help to represent addition contexts through worked examples with simple numbers and tasks with larger numbers. Three types of addition contexts are used: (1) one set of items made up of 2 parts; (2) the whole being formed from two distinct sets of items; and (3) another set of items "adding on" or "joining" to form a new whole.
Lesson 2: Subtraction of Whole Numbers
In this lesson, part-whole bar models are used to represent subtraction problems involving whole numbers. Students study various different subtraction situations (e.g. 'take away', compare) through five examples. Students then practise with further problems to consolidate this learning
Lesson 3: Multiplication & Division
Students learn how the bar model can help represent multiplication and division contexts by studying examples and practising with further tasks. The tasks encountered involve equal groups multiplication and partition and quotition variations of division problems. The examples use very simple numbers. Consolidation tasks have larger numbers and contain more mathematical information to sort through.
Lesson 4: Addition of Fractions
In this lesson, students learn how to use bar model as a tool to represent a variety of worded addition problems involving fractions, by studying worked examples and practising with further tasks. They encounter situations where the bars that make up a bar model may simultaneously represent an absolute number (e.g. $50) and a fraction of a specified quantity (quarter of a cost).
Lesson 5: Subtraction of Fractions
solve all the worked examples. Bridging material is supplied so that students

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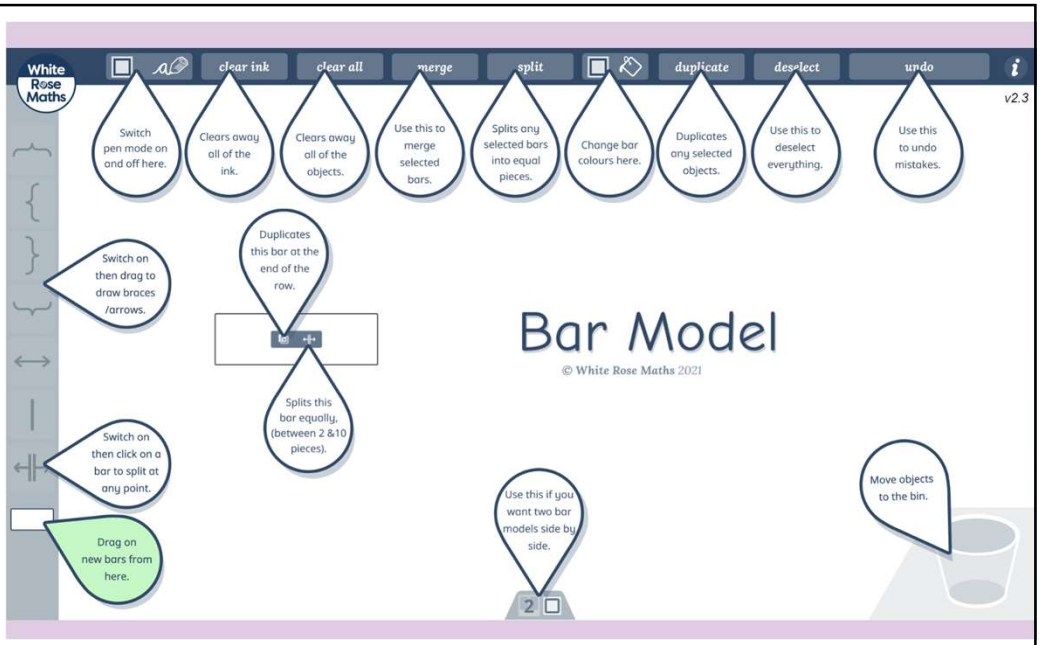
White Rose maths



https://whiteroseeducation.com/resources/barvember?

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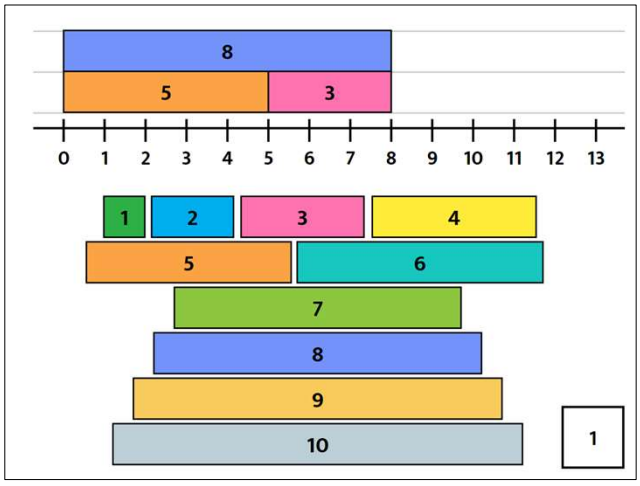
Online bar models



https://toytheater.com/number-line-bars/

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Bar model essentials

- Start in Year 1 with simple part-part-whole model and comparison models.
- Teach explicitly in Stage 1B and Stage 2 so that bar models become part of a students' 'toolbox' of problem-solving strategies.
- Encourage Stage 3 students to use when solving multi-step problems.
- Do not let students become fixated on the accuracy of their bar models
- Be aware that confident Stage 3 students may not want to use bar models, as they are able to use abstract symbols.

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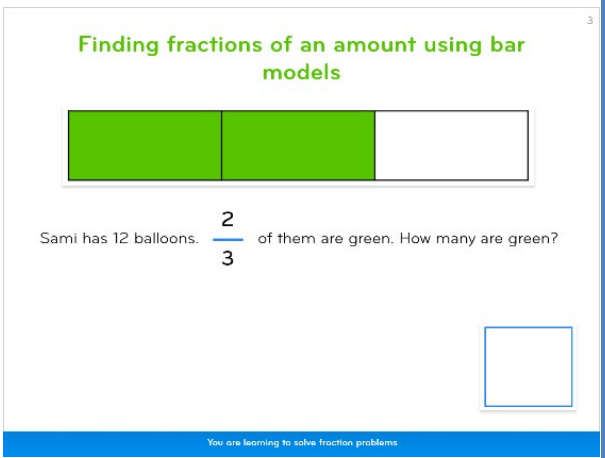

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