Making additive relations visible

Properties, principles, facts & techniques

Dr Christine Mae

Specialist: Mathematics and Numeracy K-12 Sydney Catholic Schools

Know why and how we teach Mathematics

Rationale (why)

… develop essential numeracy skills and fluency, while nurturing the ability to **think logically, critically and creatively**. They learn about **patterns** and reason about **relationships**, creating opportunities to **generalise** their solutions and to solve non-routine problems...

Working Mathematically outcome (how)

*Develop **understanding** **and** **fluency** in Mathematics **through** exploring and connecting mathematical concepts, choosing and applying mathematical techniques **to** **solve problems**, and **communicating** their **reasoning** coherently and clearly.*

NSW Mathematics K-2 Syllabus, NESA. (2021).

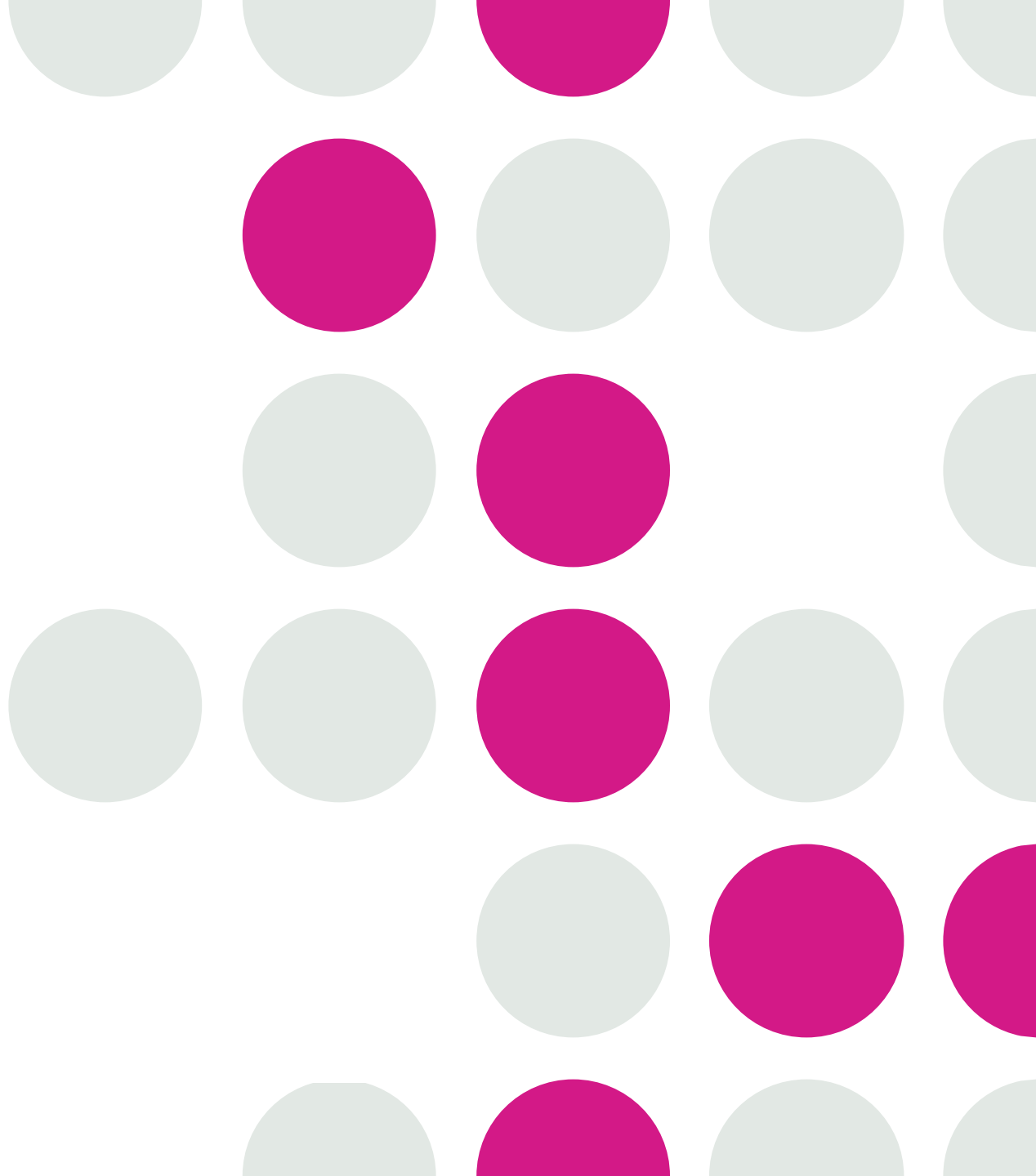
Let's make sure that students don't miss the *Mathematics* because they are too busy completing tasks.

Workshop Overview

Recognise the importance of Additive Relations.

Know important properties and principles and when they are introduced.

Consider ways to make properties and principles in Additive Relations salient for students.



The importance of Additive relations

Additive relations involves relations between numbers *and* between the processes of addition and subtraction.

Students learn that a number can be represented as itself, or as a combination of its parts. They learn to partition, regroup and rearrange numbers, and power of the principle of equivalence.

The principles, properties and relationships explored in Additive Relations underpin arithmetic strategies. They are the foundation for solving equations, numerically and algebraically.

Understanding the inverse relation between addition and subtraction, e.g. □ + 3 – 3 = □, is a strong predictor of mathematical progress.

Nunes T, Bryant P, Evans D, Bell D, Gardner S, Gardner A, and Carraher JN (2007), 'The contribution of logical reasoning to the learning of mathematics in primary school', *British Journal of Developmental Psychology*, 25:147–166, doi:10.1348/026151006X153127.

Nunes T, Bryant P, Evans D and Barros R (2015), 'Assessing quantitative reasoning in young children', *Mathematical Thinking and Learning,* 17:178–196, doi:10.1016/j.ijer.2015.09.005.

Additive and Multiplicative Reasoning

On my 10th birthday, my brother is twice my age.

- How old will my brother be on my 31st birthday?

Students need to know *how* to think additively and multiplicatively and become fluent in using related processes and tools.

Students need to know *when* to think additively and when to think multiplicatively about relations between numbers.

Students need to *think* logically, critically and creatively about patterns and generalise solutions.

Generalise solutions to identify properties

I list the numbers from 1 to 10 inclusive.

Suddenly, I realise that I can find the total of the numbers in my head by rearranging and regrouping the numbers using combinations that I already know.

- *What might be the best way to rearrange and regroup the numbers to make them easy to add mentally?*
- *How could I apply the same idea to find the total of:*
  - *the odd numbers from 1 to 20?*
  - *all numbers from 1 to 100 inclusive?*
  - *any pattern that increases or decreases by a constant amount?*

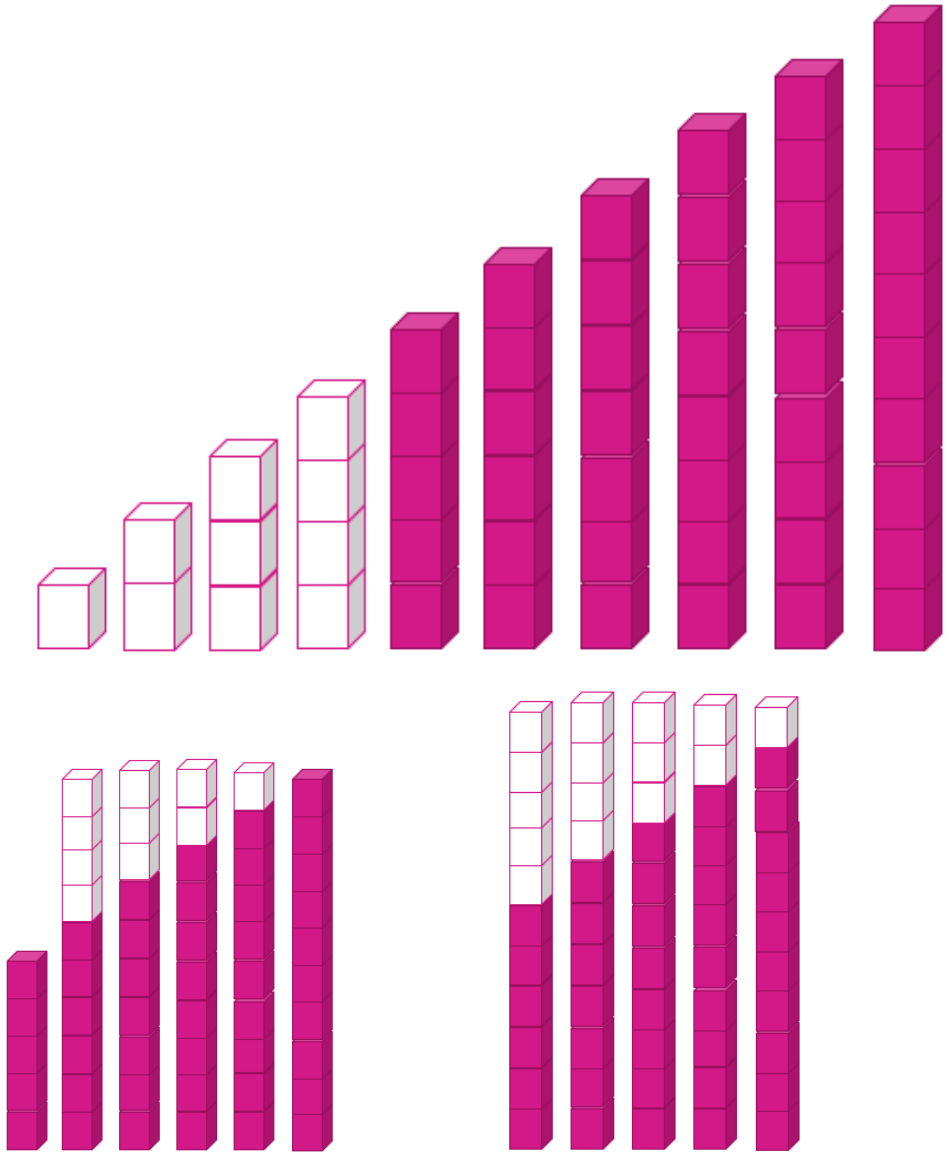
Entice students to go beyond a correct answer, to recognizing mathematical principles and communicating how and why they work (and when to select them)!

1+2+3+4+5+6+7+8+9+10 = 10 + (1+9)+(2+8)+(3+7)+(4+6)+5

1+2+3+4+5+6+7+8+9+10 = (10+1)+(9+2)+(8+3)+(7+4)+(6+5)

Use the principle of equality

- apply the **associative property** of addition to aid mental computation by forming groups of ten.



Make properties and principles salient

Connect, compare and contract properties and principles.

5 x 10 + 5 = 5 x 11

or

5 x 11 = 5 x 10 + 5 x 1

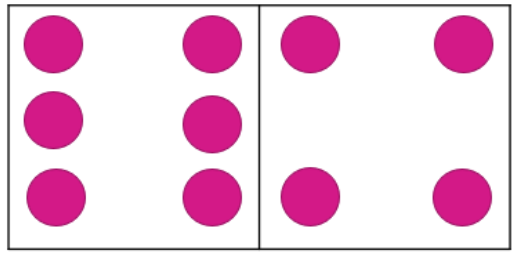
Distributive property!

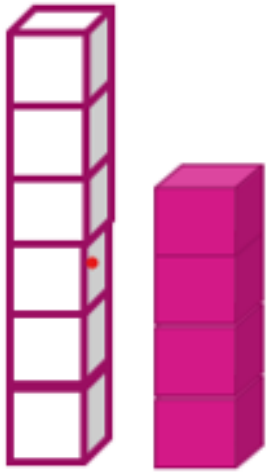
The cardinality principle and additive relations

Context: In **Early Stage 1**, students learn to count collections and find the total using one-to-one matching. They learn to combine, separate and compare collections.

Principle: when we count a collection of objects, the last number we say is the number for the collection (cardinality principle).

- Demonstrate **relations between the processes** of addition and subtraction by **combining, separating and comparing** quantities.
- Demonstrate **relations between numbers** – spatially and numerically!





Principle of equal addends (Levelling principle)

Context: In **Stage 1 Part A**, learn to use symbols to write number sentences and model, record and recall number bonds up to 10. To move from counting by ones to mental strategies, students need to ***know*** number bonds for numbers to 10.

Principle: In addition, when one addend decreases, the other addend must increase by the same amount to maintain the same total (principle of equal addends / levelling principle)!

Use patterning to reveal the levelling principle.

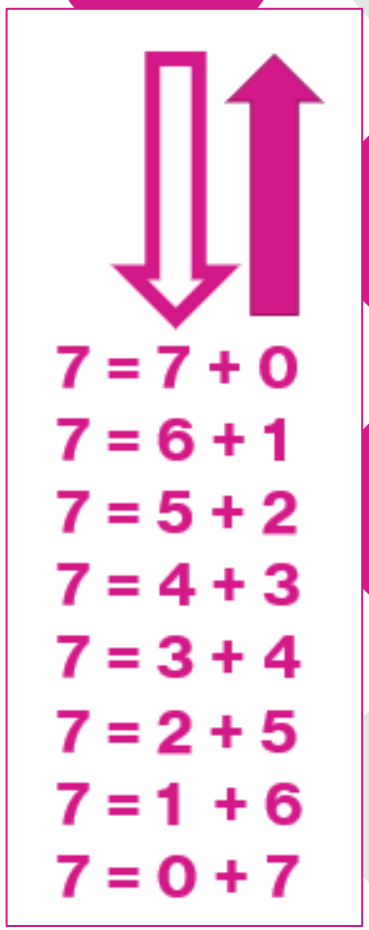
- Model and record the quantity as a combination with zero.
- Record number sentences as you move one item at a time to reveal and continue a pattern.
- Describe what happens to the other number as one number goes up/down by one.
- Recognise the power of this principle for recognising equality and as a strategy for arithmetic,

7 + 9 = 8 + 8

(make one number one more and the other number one less)!

27 + 19 = 23 + 23

(make one number four less and the other number four more)!



The commutative and inverse properties

Context: In **Stage 1 Part A**, students learn about relationships between facts.

Property: In addition, the order in which we add the numbers does not change the result (commutative property)!
Property: We can work backwards using the opposite process to subtract (inverse property)!

Move students from learning each fact separately, to recognising that they can turn facts around to reveal other facts. Teach students to start with an addition number fact, turn it around and then work in reverse using the opposite process to reveal a subtraction fact.

- Model the commutative property.
- Model the inverse relationship between addition and subtraction.
- Record 'fact families'.

5 + 2 = 7
7 – 2 + 5
2 + 5 = 7
7 – 5 = 2

2 + 5 = 7 so 5 + 2 = 7

2 + 5 = 7 so 7 – 5 = 2

The missing number could be anywhere

Solve different types of problems using the same number bond	
I start with 5 pencils. A friend gives me 4 more. *How many pencils do I have altogether?* **5 + 4 = ?**	I start with 9 pencils. I give 4 of my pencils to a friend. *How many pencils do I have now?* **9 - 4 = ?**
I start with 5 pencils. When my friend gives me some more pencils, I have 9. *How many pencils did my friend give me?* **5 + ? = 9**	I start with 9 pencils. After I give some pencils to a friend, only 5 remain. *How many pencils did I give to my friend?* **9 - ? = 5**
I have some pencils. After my friend gives me 4 more pencils, I have 9. *How many pencils did I have at the start?* **? + 4 = 9**	I start with some pencils. After I give 4 pencils to a friend, 5 pencils remain. *How many pencils did I have at the start?* **? - 4 = 5**

The principle of equivalence

Context: In **Stage 1 Part A** students use the equals sign to record equivalent number sentences involving addition. They learn that the equals sign means 'is the same as', rather than as an indication to perform an operation.

Principle: we can use relations between numbers and processes, and the principle of equivalence, to find unknown numbers.

- Teach students to read and record equivalent number sentences, using the language ‘is the same as’.
- Teach students to use objects to model equivalences.
- Teach students to use equivalence to find missing numbers.

5 and 3 is the same as 7 and 1

5 + 3 = ? + 7

Relate to principle of equal addends!

? + 5 = 10 - 2

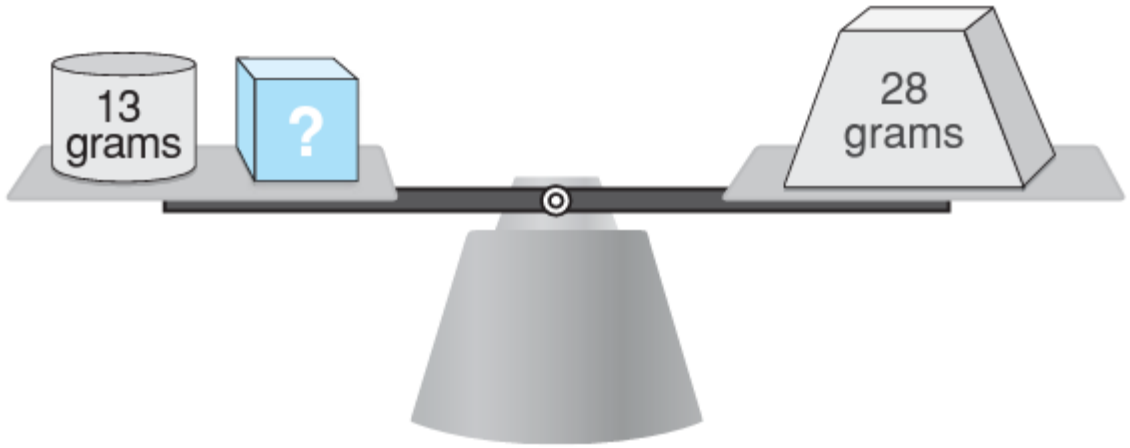
Relate to inverse property!

? + 3 = 3 + 5

Relate to commutative property!

Taking it forward

This scale is balanced.



What is the weight of the cube?

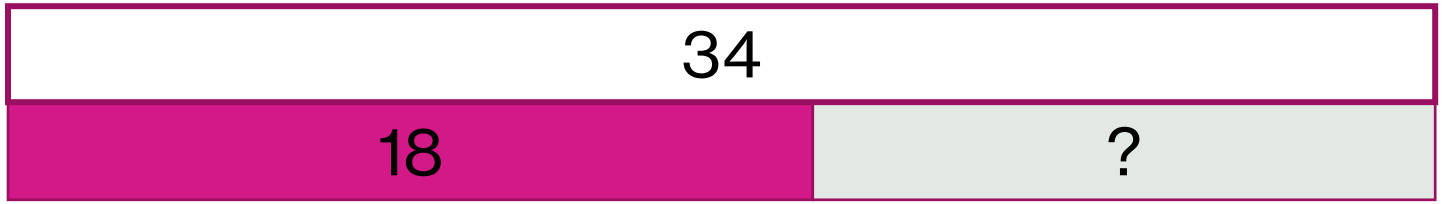
grams

- Compare the lengths of objects.
- Compare areas.
- Compare volumes.
- Compare the capacity (internal volume) of containers.
- Compare the masses of objects.
- Compare duration of events.
- Make inferences and draw conclusions about data.

The complement principle

Context: In **Stage 1 Part B**, students use the complement principle and diagrams like the bar model to represent and reason about additive relations.

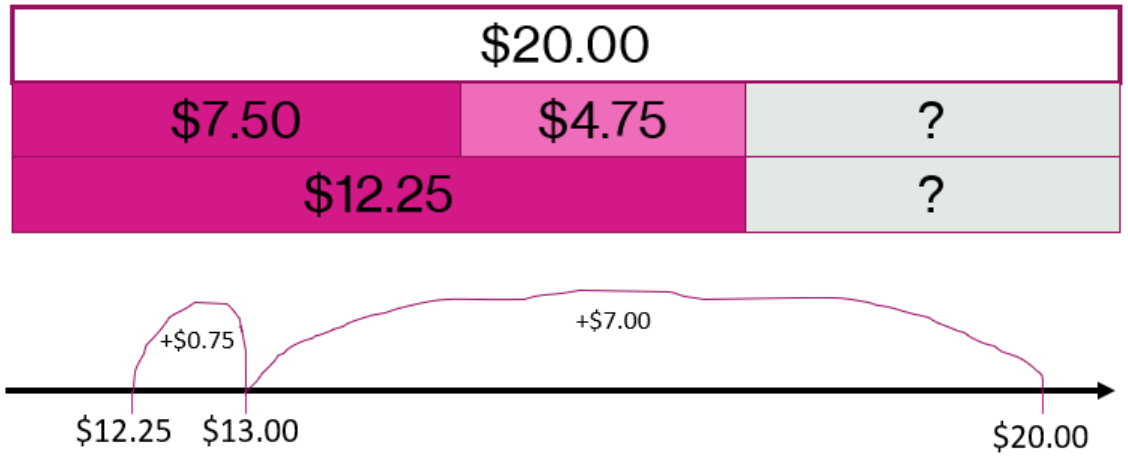
Principle: A number is the sum of its parts. We can compare a part to a whole to find the part that is missing (complement principle)!



- Teach students to draw diagrams that show the relationship between the total and the parts and how to use addition to find the difference or subtract.

Taking it forward

I purchase a sandwich for $7.50 and a drink for $4.75. How much change will I receive from $20?

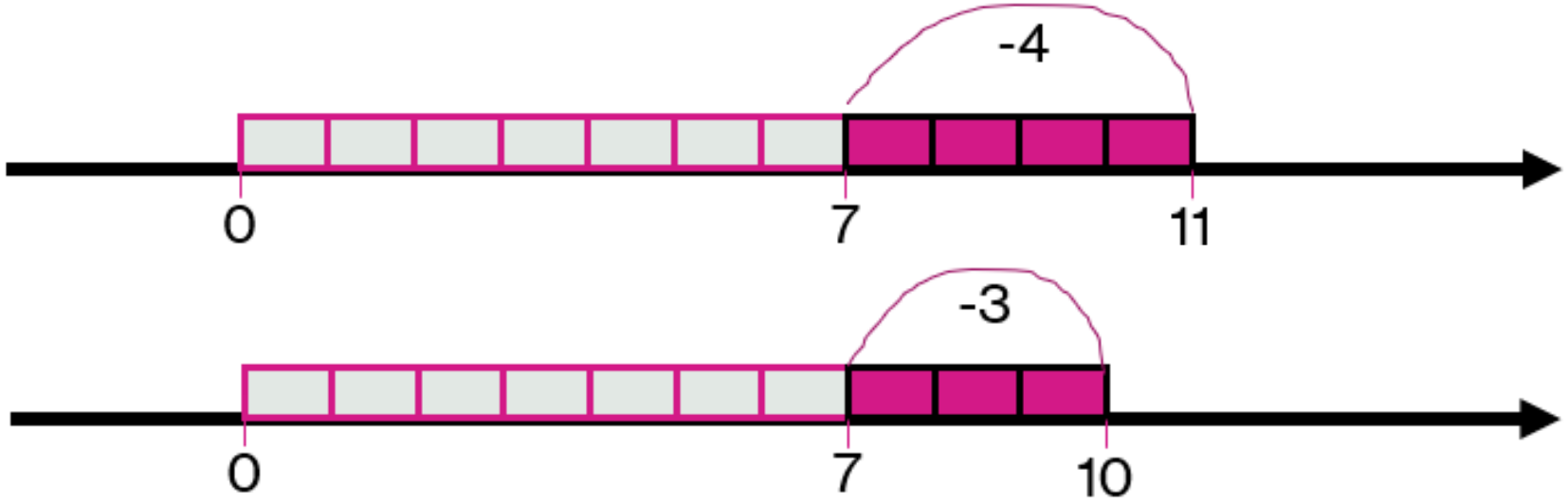


What is the difference between $2\frac{1}{8}$ and $\frac{3}{4}$?

Constant Difference Principle

Context: In **Stage 1 Part B** students represent the principle of constant difference.

Principle: In subtraction, when both numbers increase or decrease by the same amount, the difference remains the same (constant difference principle)!



51 – 27 = 24
41 – 17 = 24
31 – 7 = 24
29 – 5 = 24

- Teach students to increase or decrease the minuend (the number being taken away from) and the subtrahend (the number being taken away) by the same amount and write equivalent number sentences as a foundation for efficient subtraction strategies.

Taking it forward

10 013 - 2358

Take **14** away from each number.

$10\,013 - 2358$

$= 9999 - 2344$

1.05 – 0.78

Take **0.5** away from each number.

$1.05 - 0.78$

$= 1.00 - 0.73$

$\mathbf{3\frac{1}{8} - 1\frac{7}{8}}$

Take $\mathbf{1\frac{1}{8}}$ away from each number.

$3\frac{1}{8} - 1\frac{7}{8}$

$= 2 - \frac{6}{8}$

The Associative Property

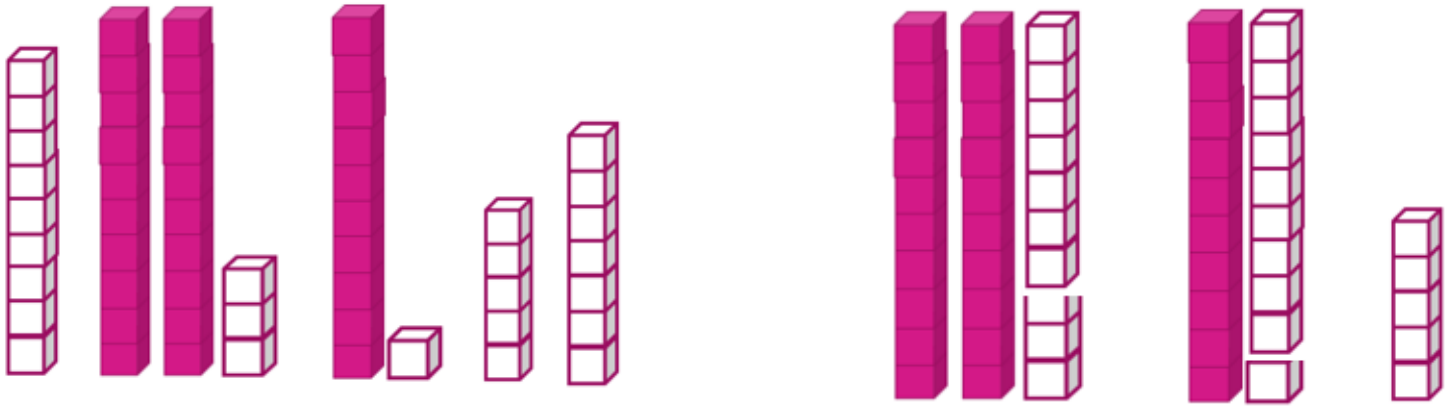
Context: In **Stage 2 Part A**, students learn to apply the associative property of addition to form multiples of 10.

Property: In addition, we can rearrange and regroup numbers without changing the total (associative property)!.

Move from looking for combinations that make 10 to looking for combinations that make multiples of 10. Model quantities and show how they can be rearranged to fit together.

- Ask students to add three or more numbers mentally.
- Teach students to mentally rearrange and regroup numbers to form multiples of ten.

9 + 23 + 11 + 5 + 7 = (23+7) + (11+9) + 5



Consider the scope of additive relations

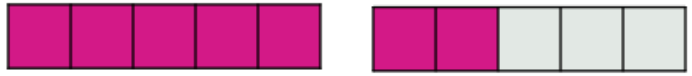
By the end of Stage 3, students recognise that numbers, including decimals, are the **sum of the Place Values** of their digits.

Additive property of Place Value

e.g. $32.125 = 30 + 2 + \frac{1}{10} + \frac{2}{100} + \frac{5}{1000}$

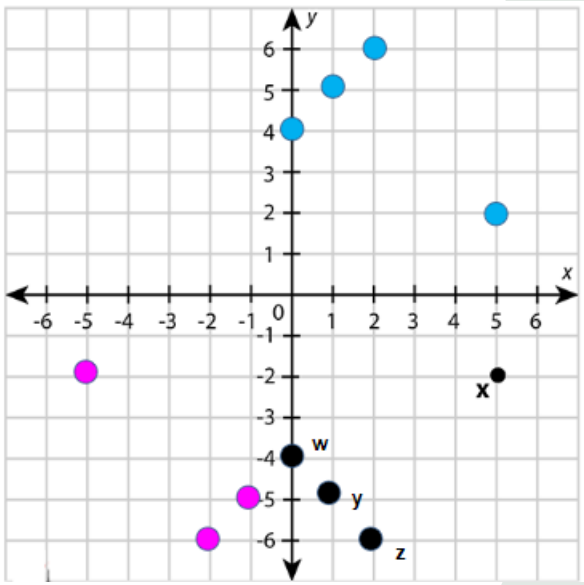
They use diagrams to subtract fractions from whole numbers, e.g. $2 - \frac{3}{5} = 1\frac{2}{5}$.

Complement principle



They predict the coordinates of a point when it is reflected across an axis on the Cartesian Plane.

Equal distance from zero



They find missing side lengths when calculating perimeter.

They use information to calculate gross and net mass.

They calculate elapsed time.

Making Additive Relations visible: properties, principles, facts and techniques.

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