



# MULTIPLICATIVE THINKING – WHAT IS IT AND WHY IS IT IMPORTANT?

A Keynote Address to the Bringing Maths to Life Conference,  
UNSW, November 2022

**By Emeritus Professor Dianne Siemon - RMIT University**

*I would like to show my respects and acknowledge the Bedegal people who are the Traditional Custodians of the Land on which this meeting takes place, and to Elders past and present*

A scar tree on the Bundoora Campus of RMIT University

# Multiplicative Thinking – a really BIG IDEA

## TAIL STRIKE ...

On 20 March 2009, an Emirates Airbus 340-500 with 257 passengers, 12 cabin crew and 4 flight crew safely returned to Melbourne Airport after a tail strike on take-off.

The investigation found that the take-off weight of the aircraft entered into the flight computer was 100 tonnes less than the actual weight ...an error of 1 in the hundreds place!



# DNA EVIDENCE ...

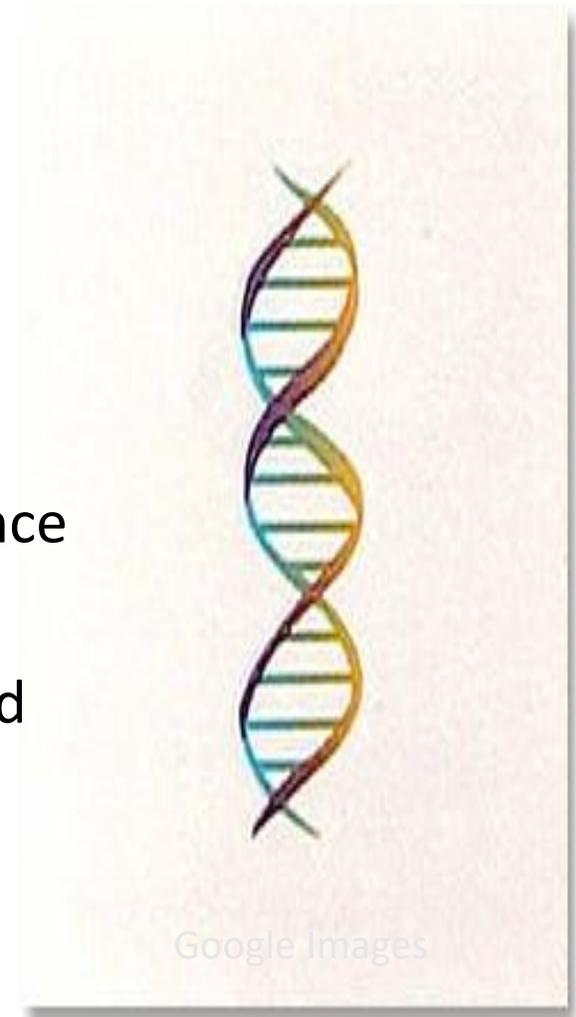
Some years back, the American Jury Association conducted a mock trial to explore some of the issues involved in the use of DNA evidence.

Two juries were convened to hear exactly the same evidence with one difference:

Jury 1 was told that there was a 2% chance that the DNA evidence found at the crime site did not come from the accused.

Jury 2 was told that the probability that the DNA evidence found at the crime site did not come from the accused was 0.02.

The conviction rates were significantly different!





Google Images

## MEDICATION ...

A seriously ill colleague was taken to hospital and lapsed into a coma.

A day or so later he was semi-conscious but unable to move or speak ...

He was aware that he was about to be medicated but became concerned when he heard two nurses debating as to whether or not a 1% solution meant 1 mL in 10 mL or 1 mL in 100 mL ...

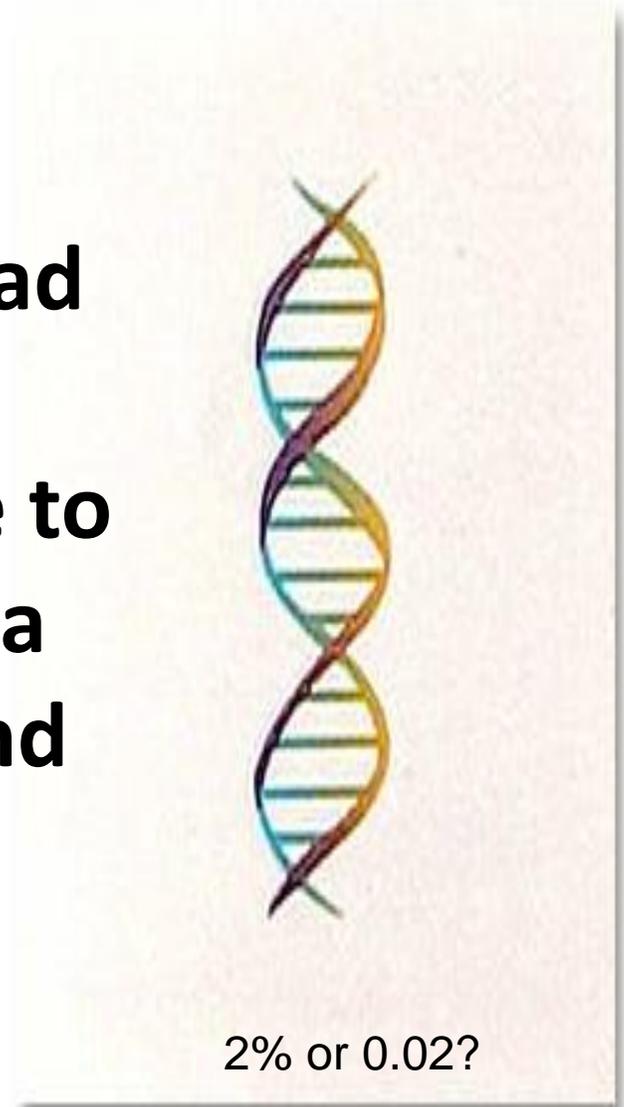


Actual: 362.9 tonnes  
Entered: 262,9 tonnes

1%?  
1 in 10 or  
1 in 100?



**The ability to read  
and interpret  
numbers relative to  
context can be a  
matter of life and  
death!**



2% or 0.02?

# Taught but not learnt?

Year 5: Compare, order, and represent decimals (ACARA)

Year 6: Make connections between equivalent fractions, decimals, and percentages (ACARA)

Year 8: Solve problems involving the use of percentages ... (ACARA)

**Reading and interpreting numbers relative to context requires more than discrete skills and competencies! It requires:**

- A deep understanding of the base ten system of numeration
- Recognition that fractions, decimals, and percentages are just different ways to represent a multiplicative relationship between two quantities.
- An alertness to context, a critical stance, and an inclination to check for reasonableness

# A significant issue

Only 56% of Australian 15-year-olds solved this PISA problem in 2012?

You are making your own dressing for a salad.

Here is a recipe for 100 millilitres (mL) of dressing.

Salad oil: 60 mL

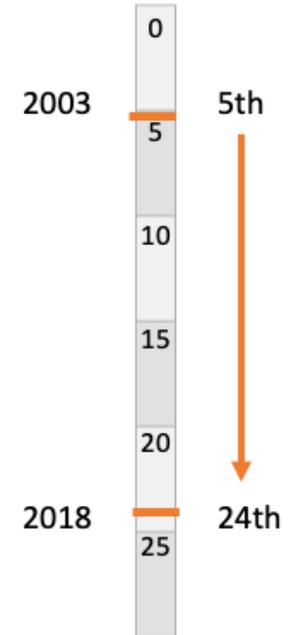
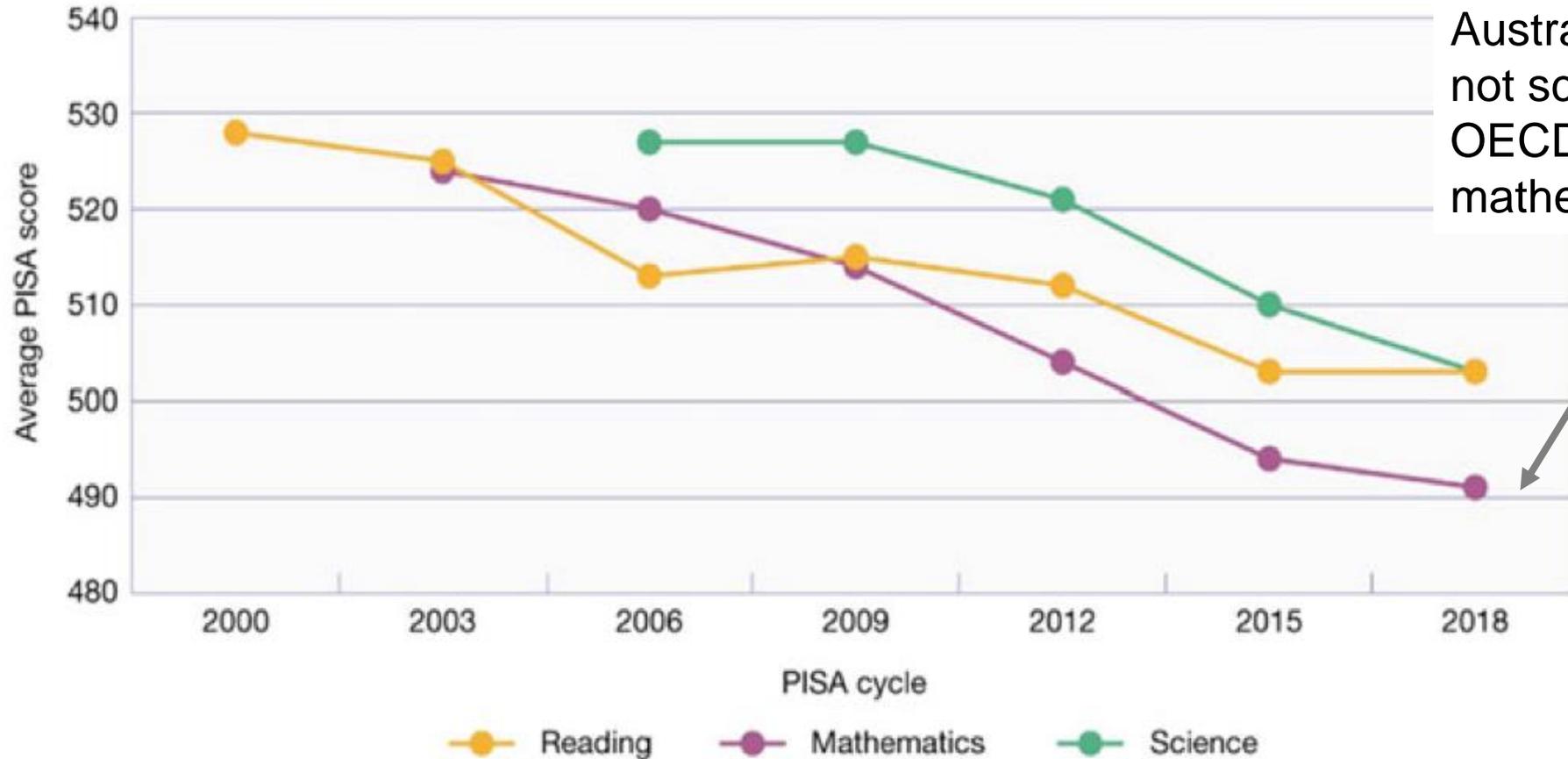
Vinegar: 30 mL

Soy Sauce: 10 mL.

How many millilitres (mL) of salad oil do you need to make 150 mL of this dressing?

# What we are doing is not working

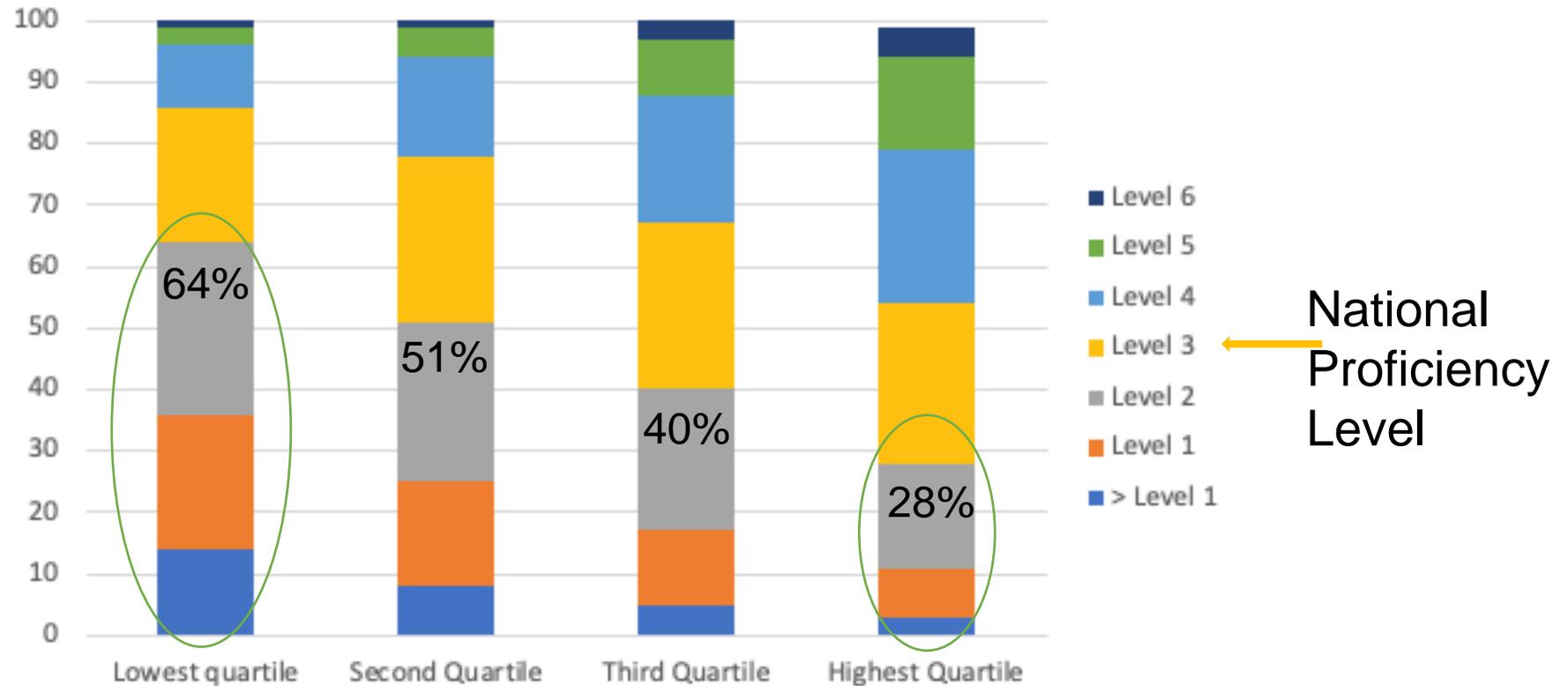
2018 is the first time Australian students have not scored above the OECD average in mathematical literacy



*Australian students' performance in PISA 2000-2018*

Source: Thomson, S. (2021). *Australia: PISA Australia – Excellence and Equity?* (p. 31). ACER <https://research.acer.edu.au/cgi/viewcontent.cgi?article=1052&context=ozpisa>

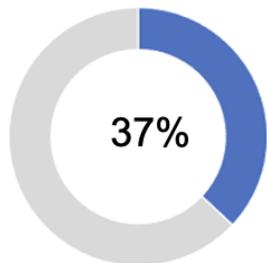
Students from lower socioeconomic backgrounds are more than twice as likely to be below the National Proficiency Level than students from higher socioeconomic backgrounds



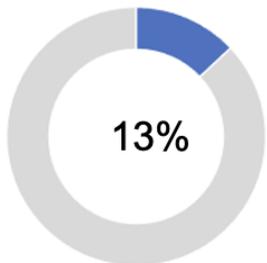
*Percentage of students across mathematical proficiency levels by socioeconomic background PISA 2018 (Source: Thomson, 2021)*

# Participation in most Year 12 **maths** and **science** subjects is declining and for **science** it is the **lowest in 20 years**

Australian Office of the Chief Scientist. (2017).  
<https://www.chiefscientist.gov.au/sites/default/files/2-Science-and-Maths-in-Australian-Secondary-Schools-datasheet-Web.pdf>



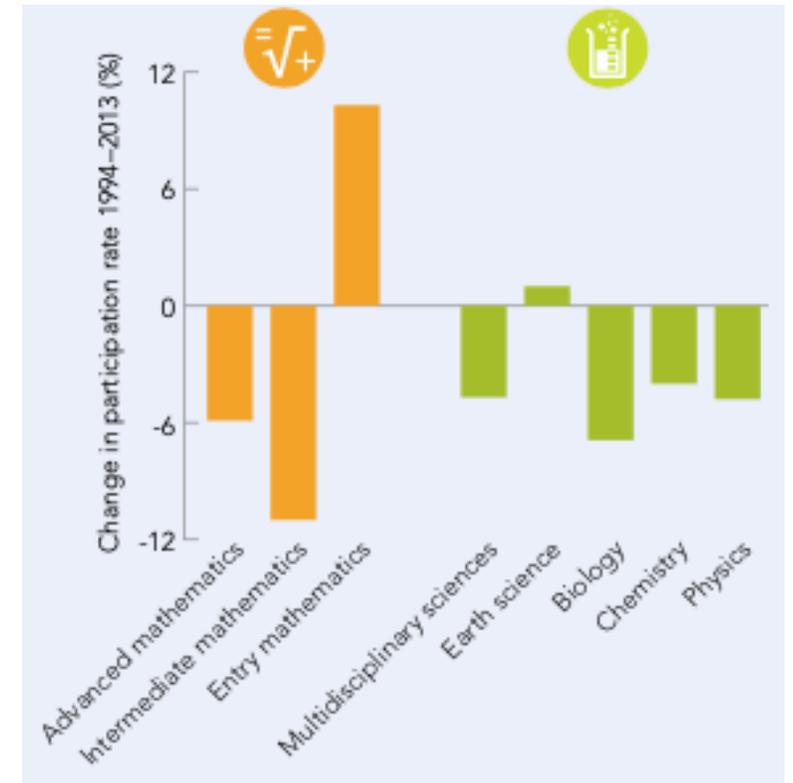
Year 4



Year 8

## Students are losing interest in school **mathematics** at an early age

Thompson et al., 2017. TIMSS2015 – Reporting Australia’s Results, ACER



# What can we learn from research?

*... excellence in teaching is the single most powerful influence on student achievement (Hattie, 2003, p. 4)*

*... well-grounded evidence of pupils' progressions in learning is crucial to the work of teachers (Black, Wilson, & Yeo, 2011, p. 71)*

*... assessment for learning, is one of the most powerful ways of improving student achievement (William, 2013, p. 15)*

## Assessment for learning

*An assessment functions formatively to the extent that evidence about student achievement is **elicited, interpreted, and used** by teachers, learners, or their peers to make decisions about the next steps in instruction that are likely to be better, or better founded, than the decisions they would have made in the absence of that evidence.*

(William, 2011, p. 43, my emphasis)

# It's not rocket science

A social-constructivist view of learning requires that we recognise and act on three key processes in learning. An understanding of:

- where the learner is right now;
- where the learner needs to be; and
- how to get there.

William, D. (2013). Assessment: The bridge between teaching and learning.  
*Voices from the Middle*, 21(2), 15 – 20

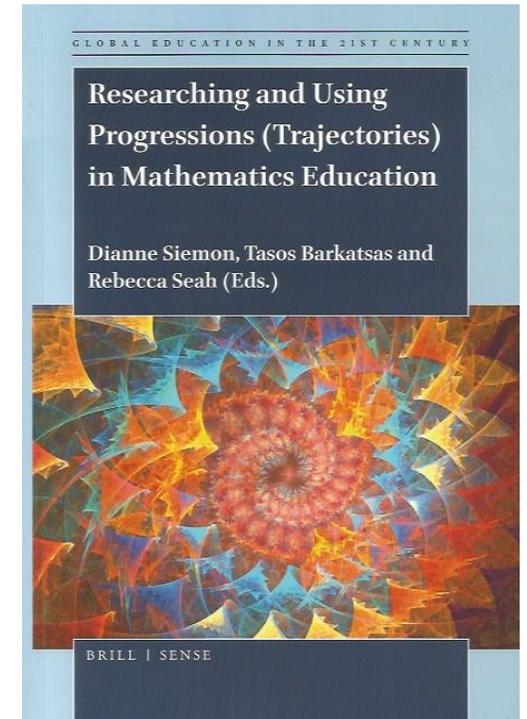
But **where** in relation to what?

**Year level curriculum expectations or what research suggests is most likely to make a difference?**

# Learning Progressions

*In recent years attention has turned to the **development of evidenced-based learning trajectories (or progressions)** as a means of identifying what mathematics is important and how it is understood over time.*

*But for this information to be useful to practitioners, it needs to be accompanied by **accurate forms of assessment** that locate where learners are in their learning journey and **evidenced-based advice** about where to go to next.  
(Siemon, 2019, p.6)*



Siemon, D. (2019). Knowing and building on what students know: The case of multiplicative thinking. In D. Siemon, T. Barkatsas & R. Seah (Eds.), *Researching and Using Progressions (Trajectories) in Mathematics Education* (pp. 6-31). Brill-Sense

**Middle Years Numeracy Research Project (MYNRP, 1999-2001)** - explored number sense, measurement & data sense, and spatial sense in a structured sample of just under 7000 students in Years 5 to 9 students using rich tasks and Rasch modelling

## Numeracy

- core mathematical knowledge - in this case, number sense, measurement and data sense and spatial sense as elaborated in the *National Numeracy Benchmarks for Years 5 and 7* (1997);
- the capacity to critically apply what is known in a particular context to achieve a desired purpose; and the
- actual processes and strategies needed to communicate what was done and why.

(Siemon & Virgona, 2001)

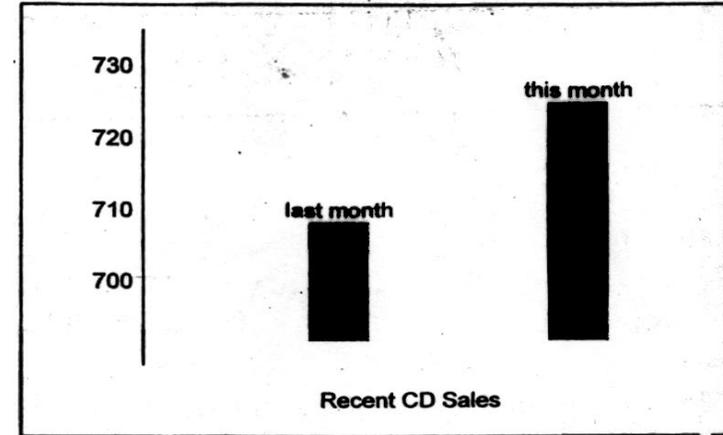
# A rich task and scoring rubric\*

0. No response or 'yes' or 'no' without a reason.
1. Reasoning based on numbers alone, no recognition that 'big' is relative.
2. Reasoning shows some recognition that 'big' is relative to total sales, but unsupported conclusion, little/no explanation (e.g., "it depends ...").
3. Reasoning concludes that increase is not 'big' relative to total sales, some attempt to relate this to proportion (e.g., "15 out of 725 is not very big").
4. Correct conclusion, "not big", %, fractions, ratio used correctly to support well-reasoned explanation.

\* Task adapted from a PISA item, response from MYNRP

## CD SALES

The manager of a Music shop showed this graph and said "There's been a big increase in the number of CD sales this month."



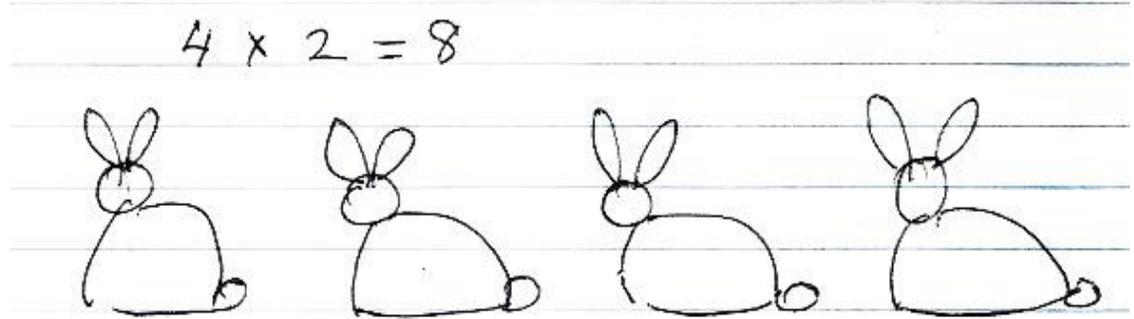
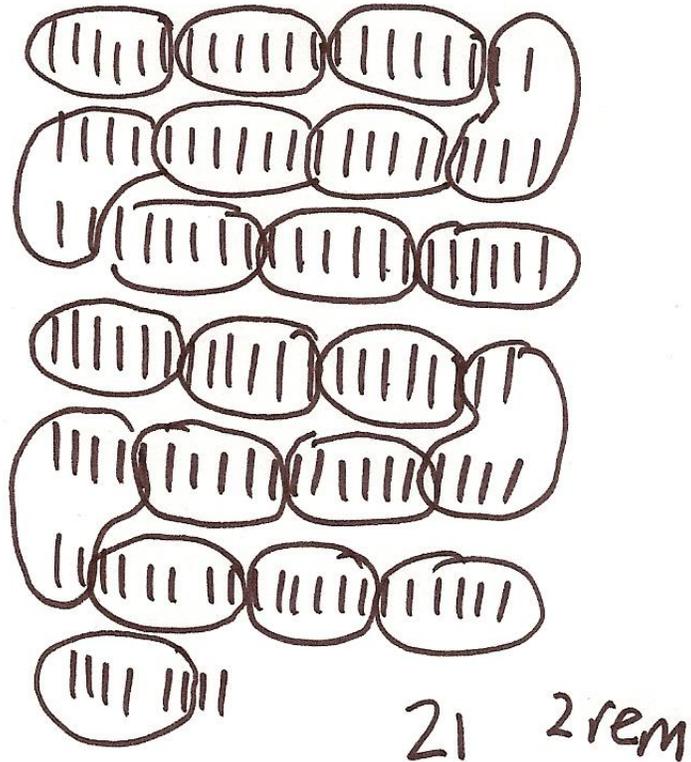
Do you consider the Manager's statement to be a reasonable interpretation of the graph? Explain your reasoning.

No because the increase has been exaggerated.



If you were to go from zero you would see that the increase isn't very large. The graph above only shows the tip of the graph ~~is~~ making it look bigger.

A Year 8 student's response to the question: What sort of maths do you really like doing?



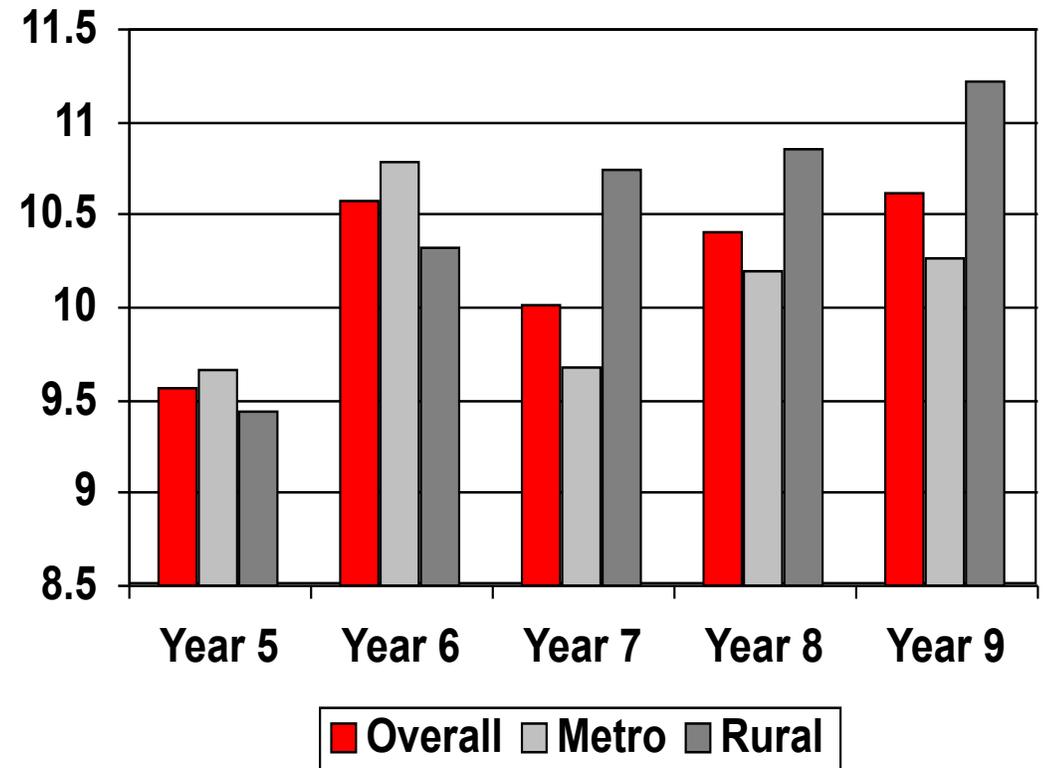
A Year 7 student's response to the problem:  $128 \div 6 = \square$

Suggests understanding limited to the **equal groups** idea for multiplication and the corresponding **quotition** idea for division

Two responses from the 'At Risk' Individual Student Interviews, MYNRP (2000)

# MYNRP Results

- There is a 5 to 7-year range in student mathematics achievement in each year level from Years 5 to 9
- There is as much variation within schools as between schools (individual teachers make a significant difference to student learning);
- The needs of many students, but particularly those 'at risk' or 'left behind', were not being met.
- Differences in performance were almost entirely due to an inadequate understanding of fractions, decimals, and proportion (i.e., **multiplicative thinking**), and a reluctance/inability to explain/justify solutions.



Mean Adjusted Logit Scores by Location November 1999 (N=6879)

***Scaffolding Numeracy in the Middle Years (SNMY) Project (2004-2006)*** - explored the development of multiplicative thinking in a purposeful sample of just over 3200 Year 4 to 8 students using rich tasks and Rasch modelling

**Multiplicative Thinking** defined in terms of:

- a capacity to work flexibly and efficiently with an extended range of numbers (for example, larger whole  $\frac{L}{SEP}$  numbers, decimals, common fractions, ratio, and per cent);
- an ability to recognise and solve a range of problems involving multiplication, division and/or proportional reasoning; and
- the means to represent and communicate this effectively in a variety of ways (e.g., words, diagrams, symbolic  $\frac{L}{SEP}$  expressions, and written algorithms),

(Siemon, Breed, Dole et al, 2006)

# Multiplicative thinking is fundamentally different to additive thinking

A fruit stall sold 14 punnets of strawberries. If they had 45 punnets to start with, how many did they have left for sale?

Part	Part
Whole	

Additive  
Structure

12 strawberry plants per row, 18 rows, how many plants?

Multiplicative  
Structure

$M_1$	$M_2$
1	12
18	?

## Multiplicative thinkers:

- are **alert to and reason with relationships between different quantities** (e.g., distance and time, kilograms and minutes)
- recognise and **use the *for each* and *factor-factor-product* ideas** to solve problems,
- no longer rely on *repeated addition* or a count of equal groups.

Because simple multiplicative problems can be solved additively by repeated addition or a learnt algorithm and known facts, **it can be difficult to determine whether or not a student is thinking multiplicatively.**

# SNMY Extended Task

## BUTTERFLY HOUSE...



Some children visited the Butterfly House at the Zoo.



They learnt that a butterfly is made up of 4 wings, one body and two feelers.

While they were there, they made models and answered some questions.

**For each question, explain your working and your answer, in as much detail as possible.**

- a. How many wings, bodies and feelers would be needed for 7 model butterflies?

\_\_\_\_\_ wings  
\_\_\_\_\_ bodies  
\_\_\_\_\_ feelers

- b. How many complete model butterflies could you make with 16 wings, 4 bodies and 8 feelers?

This task had 9 items altogether including:

Items of **increasing complexity**, eg, “How many complete model butterflies could you make with 29 wings, 8 bodies and 13 feelers?”

Items involving **simple proportion and rate**, eg, “To feed 2 butterflies, the zoo needs 5 drops of nectar per day. How many drops would be needed per day to feed 12 butterflies?” ...and

Items involving the **Cartesian product**, eg, given 3 different body colours, 2 types of feelers and 3 different wing colours, “How many different model butterflies could be made?”

Adapted from ‘Butterflies and Caterpillars’ (Kenney, Lindquist & Heffernan, 2002) for the SNMY Project (2003-2006)

# SNMY Short Task

## ADVENTURE CAMP ...



Camp Reefton offers 4 activities. Everyone has a go at each activity early in the week. On Thursday afternoon students can choose the activity that they would like to do again. The table shows how many students chose each activity at the Year 5 camp and how many chose each activity at the Year 7 camp a week later.

	Rock Wall	Canoeing	Archery	Ropes Course
Year 5	15	18	24	18
Year 7	19	21	38	22

Camp Reefton Thursday Activities

- What can you say about the choices of Year 5 and Year 7 students?
- The Camp Director said that canoeing was more popular with the Year 5 students than the Year 7 students. Do you agree with the Director's statement? **Use as much mathematics as you can to support your answer.**

Open-ended question

Problem solving, solution strategy unclear

Reading and interpreting quantitative data relative to context

Recognising relevance of proportion

Mathematics used (e.g., percent, fractions, ratio)

SNMY Project (2003-2006)



### ADVENTURE CAMP ...

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Camp Reefton Thursday Activities

a. What can you say about the choices of Year 5 and Year 7 students?

$$\begin{array}{r} 215 \overline{) 19} \\ 18 \quad 21 \\ \underline{24} \quad 38 \\ 18 \quad 22 \\ \underline{75} \quad 100 \end{array}$$

The majority of both groups chose Archery while the other activities have around the same numbers except for the rock wall which more year 7s chose

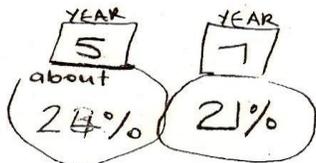
b. The Camp Director said that canoeing was more popular with the Year 5 students than the Year 7 students. Do you agree with the Director's statement? Use as much mathematics as you can to support your answer.

$$\begin{array}{r} 215 \overline{) 19} \\ 18 \quad 21 \\ \underline{24} \quad 38 \\ 18 \quad 22 \\ \underline{75} \quad 100 \end{array}$$

The director is probably right because all together there are more year 7s than 5s so that the percentage of 5s would be higher than 7s

$$\frac{25}{75} = \frac{18}{6}$$

$$4 \overline{) 25} = 6.25$$



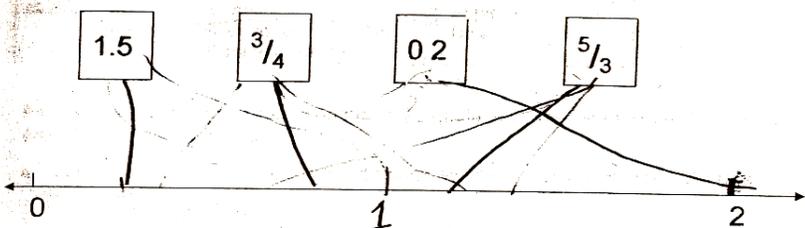
### ADVENTURE CAMP ...

TASK:	RESPONSE:	SCORE
a.	No response or incorrect or irrelevant statement	0
	One or two relatively simple observations based on numbers alone (e.g., "Archery was the most popular activity for both Year 5 and Year 7 students", "More Year 7 students liked the rock wall than Year 5 students")	1
	At least one observation which recognises the difference in total numbers (e.g., "Although more Year 7s actually chose the ropes course than Year 5, there were less Year 5 students, so it is hard to say")	2
b.	No response	0
	Incorrect (No), argument based on numbers alone (e.g., "There were 21 Year 7s and only 18 Year 5s")	1
	Correct (Yes), but little/no working or explanation to support conclusion	2
	Correct (Yes), working and/or explanation indicates that numbers need to be considered in relation to respective totals (e.g., "18 out of 75 is more than 21 out of 100"), but no formal use of fractions or percent or further argument to justify conclusion	3
	Correct (Yes), working and/or explanation uses comparable fractions or percents to justify conclusion (e.g., "For Year 7 it is 21%. For Year 5s, it is 24% because $18/75 = 6/25 = 24/100 = 24\%$ ")	4
		23

MISSING NUMBERS ...

24.

a. These numbers have been left off the number line. Without using a ruler, draw lines from each fraction to the number line below to show where it belongs. Try to be as accurate as you can.



b. For each fraction explain why you located it where you did.

1.5 I located it where I did because I knew that 1.5 was less than all of the others and I thought before I did it

$\frac{3}{4}$  I put it where I did because I knew that added up it would be 7 with is 1 more than 6 with 1.5 added up to and I knew that it would be equal

0.2 I put it where I did because it was the biggest number and it

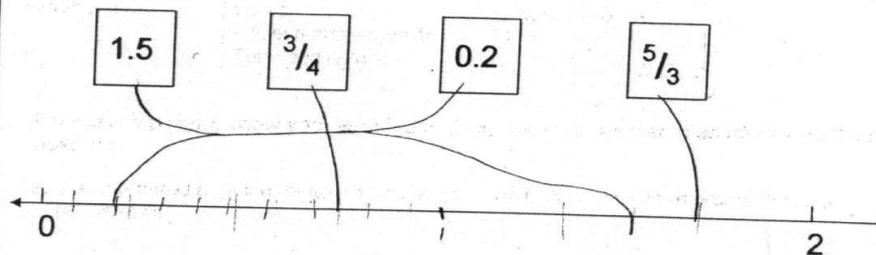
$\frac{5}{3}$  I put it where I did because I knew that five is more than 3 and 3 is more than 1

S3 - SCAFFOLDING NUMERACY IN THE MIDDLE YEARS LINKAGE PROJECT - RMIT University, May 2004

MISSING NUMBERS ...

22.

a. These numbers have been left off the number line. Without using a ruler, draw lines from each fraction to the number line below to show where it belongs. Try to be as accurate as you can.



b. For each fraction explain why you located it where you did.

1.5 I know that .5 means half and 1. means full so it is one and a half

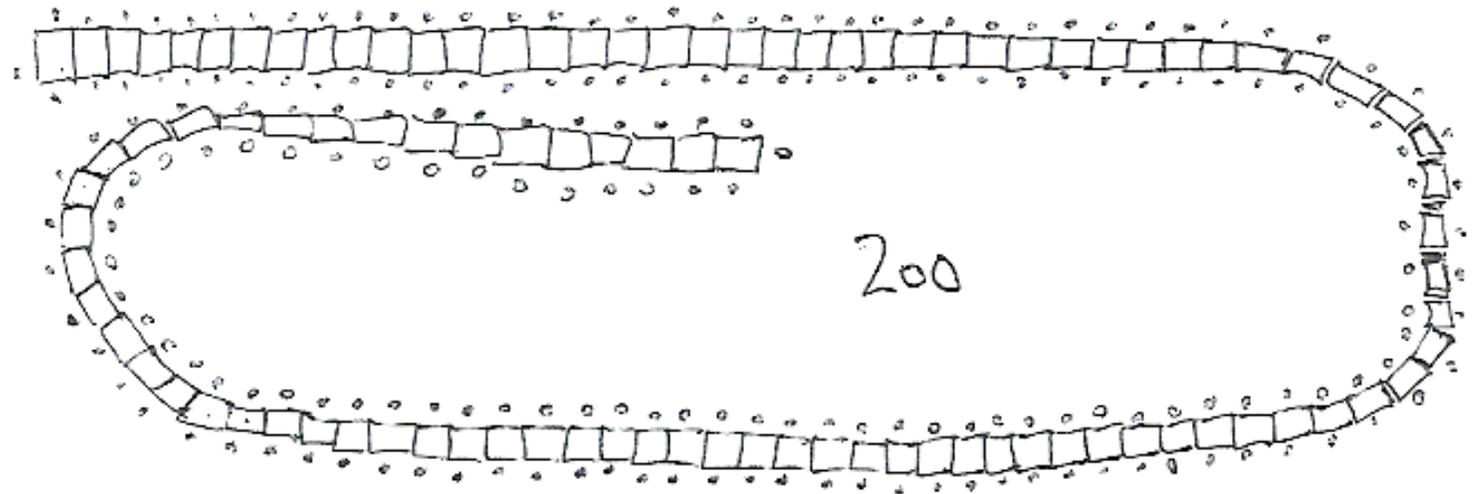
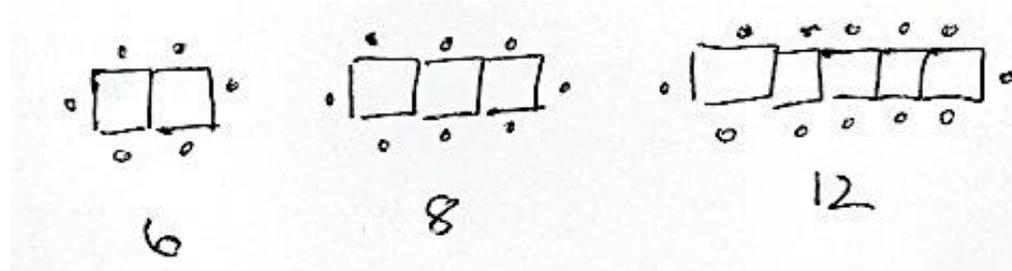
$\frac{3}{4}$  I divided 0-1 into 4 quarters and put the line on the 3rd dash

0.2 I divided 0-1 into equal parts then counted 2 and made the line join with that

$\frac{5}{3}$  In one there are 3 thirds so I divided 1-2 into 3 parts and counted on an extra 2 because 2+3

# Naïve Response to Tables and Chairs Extended Task:

20% Year 4 and close to 6% Year 8 students drew a diagram (i.e., made and counted all) to find number of people sitting at 99 tables

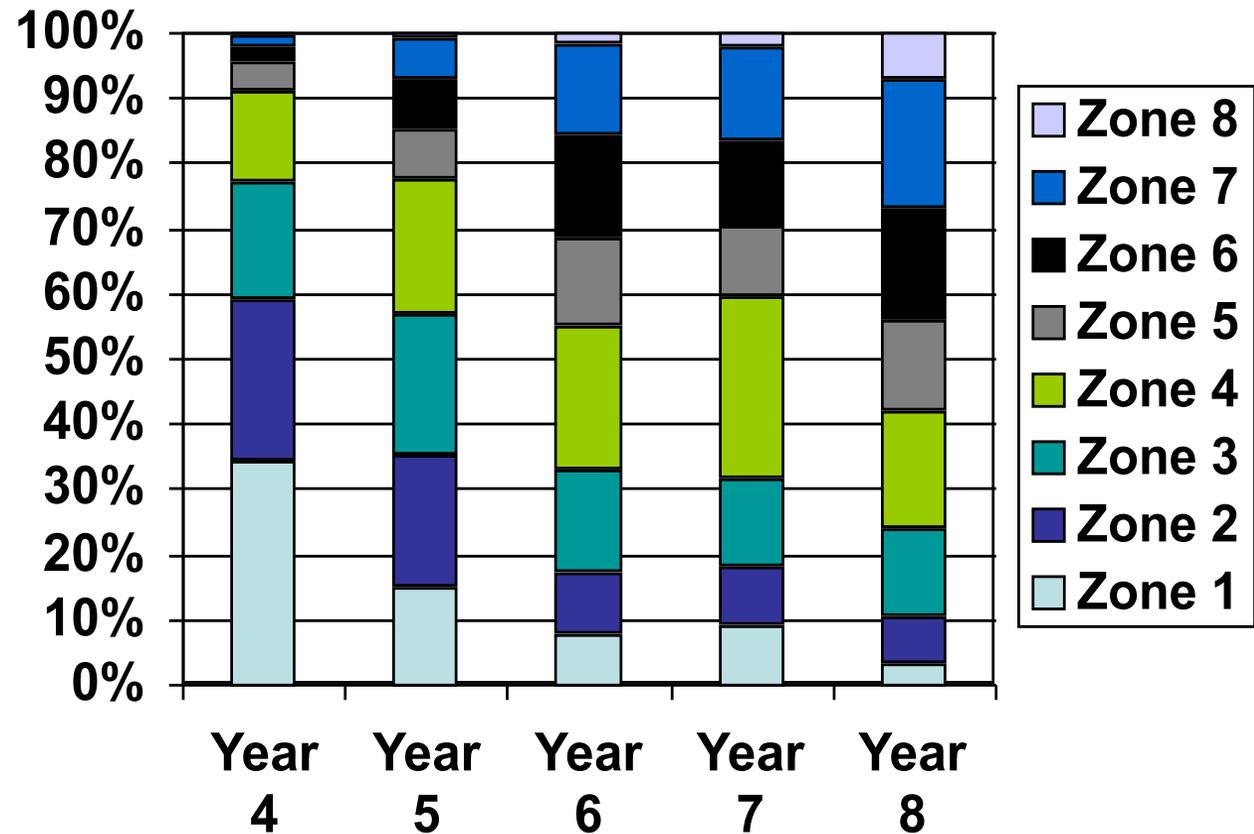


Scaffolding Numeracy in the Middle Years Research  
Project 2004-2006



## Results ... Access to Multiplicative thinking confirmed as the reason for the 6-7-year range in each year level.

Zone 4 of the LAF can be viewed as a transitional zone from additive to multiplicative thinking, suggesting that about **40% of Year 7 and 30% of Year 8 students might be deemed to be 'left behind'** in terms of curriculum expectations ... (Siemon et al, 2006)



*Proportion of Students at each Zone of the LAF by Year Level, Initial Phase of SNMY, May 2004 (N=3169)*

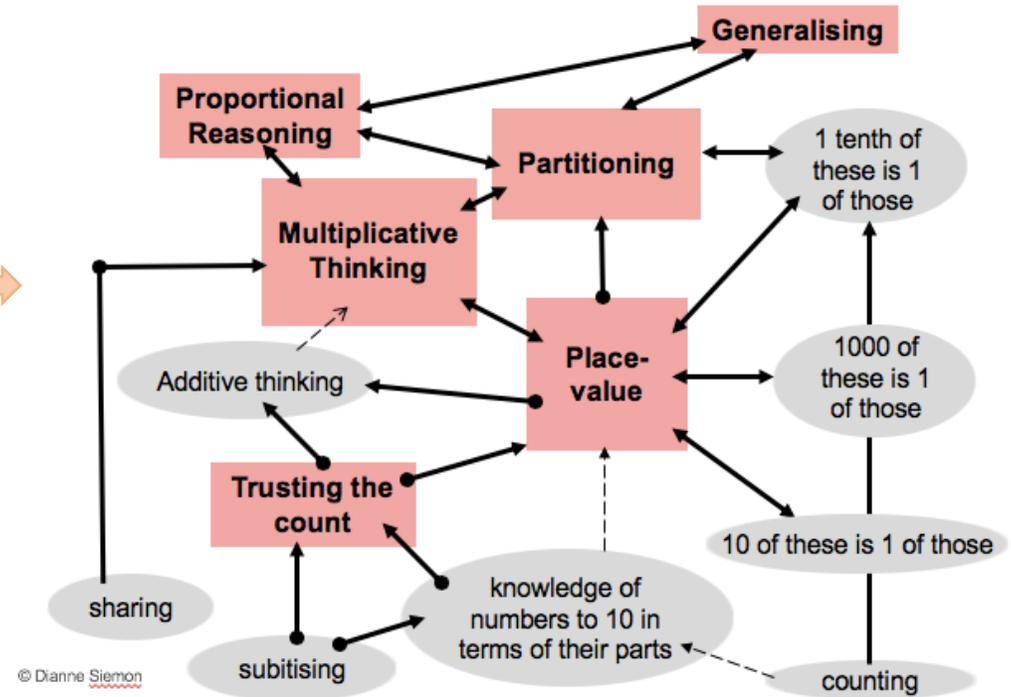
# The Big ideas in Number

LAF Zone	Level Description
8	Solves and justifies a wide range of problems involving unfamiliar <u>multiplicative situations</u> including fractions and decimals, solves complex <u>proportional reasoning</u> problems, formally describe patterns in terms of general rules, solves complex, open-ended problems
7	Compare, order, sequence, represent, and rename whole numbers, fractions, decimals, integers, inverse and identity relations, structure of place value system, recognise, describe and apply relationships between variables, algebraic processes, ratio,
6	Extend decimal fractions, use partition processes for dealing with all four operations, notion of variable, formally describe p
5	Uses <u>partitioning strategies</u> locate a value larger whole numbers and tenths multiplying and dividing, area idea, Cartesian product problems, factors and multiples, strategies for adding and subtracting unlike fractions
4	More efficient strategies for multiplying and dividing larger whole numbers, Tenths as a new place-value part, use partitioning strategies to compare fractions, times as many idea
3	<u>Place-value</u> based strategies, simple proportion problems, Cartesian product, <u>thirding and fifing</u> partitioning strategies, key fraction generalisations, works with simple patterns
2	More efficient strategies for counting large collections, array/region-based strategies for multiplication facts, halving partitioning strategies to create fraction representations, key fraction generalisations, extended place value
1	<u>Trust the count</u> (part-part-whole knowledge, addition and subtraction, 2 and 3-digit place arrays and regions)

Item analysis pointed to a small number of **BIG IDEAS** in Number that needed to be developed and consolidated over time



Assessment for Common Misunderstandings F-10 (AfCM, 2006)



The Learning Assessment Framework for Multiplicative Thinking (SNMY 2004-2006)

# What is a 'big idea' in this context?

- An idea, strategy, or way of thinking about some key aspect of mathematics, **without which students' progress in mathematics will be seriously impacted** (e.g., *trusting the count, place value*)
- Encompasses and **connects many other ideas** and strategies (e.g., *multiplicative thinking*)
- Provides an **organising structure** or a frame of reference that supports further learning and generalisations (e.g., *place value, multiplicative thinking, partitioning*)
- Cannot be completely defined but can be **observed in activity** (e.g., *partitioning, proportional reasoning, generalising*).

(Siemon 2006, 2011)

# Targeted teaching

Is a form of differentiation that is specifically concerned with addressing students' learning needs in relation to a **small number of really 'big ideas' in Number without which students' progress in school mathematics will be seriously impacted.**

*Assessment for Common Misunderstandings* (Siemon, 2006)

Goss, P., Hunter, J., Romanes, D., & Parsonage, H. (2015).  
*Targeted teaching: how better use of data can improve student learning.* Grattan Institute

GRATTAN  
Institute

July 2015



# Targeted Teaching requires:

- **assessment techniques** that expose student thinking in relation to specific aspects of each big idea;
- **a clear understanding of the key mathematical ideas, representations and strategies** that are likely to make a difference, how they are developed over time, and how they are connected;
- **interpretations of what those responses might mean and practical suggestions** about where to start teaching

(Siemon, 2006, 2011)

**Valid  
assessment  
tools**

**Evidenced-  
based Learning  
Progressions**

**Targeted  
Teaching  
Advice**

## For teachers and school leaders this requires:

- a **commitment to undertake and act on the evidence** to inform both in-the-moment and future teaching to better target the learning needs of all students;
- an **expanded repertoire of teaching approaches** that accommodate and nurture discourse, help uncover and explore student's ideas, and ensure all students can participate in the enterprise;
- **sufficient time** with students to develop trust and supportive relationships; and
- **flexibility** to spend time with those who need it most.

(Siemon & Virgona, 2001)

## Targeted teaching works

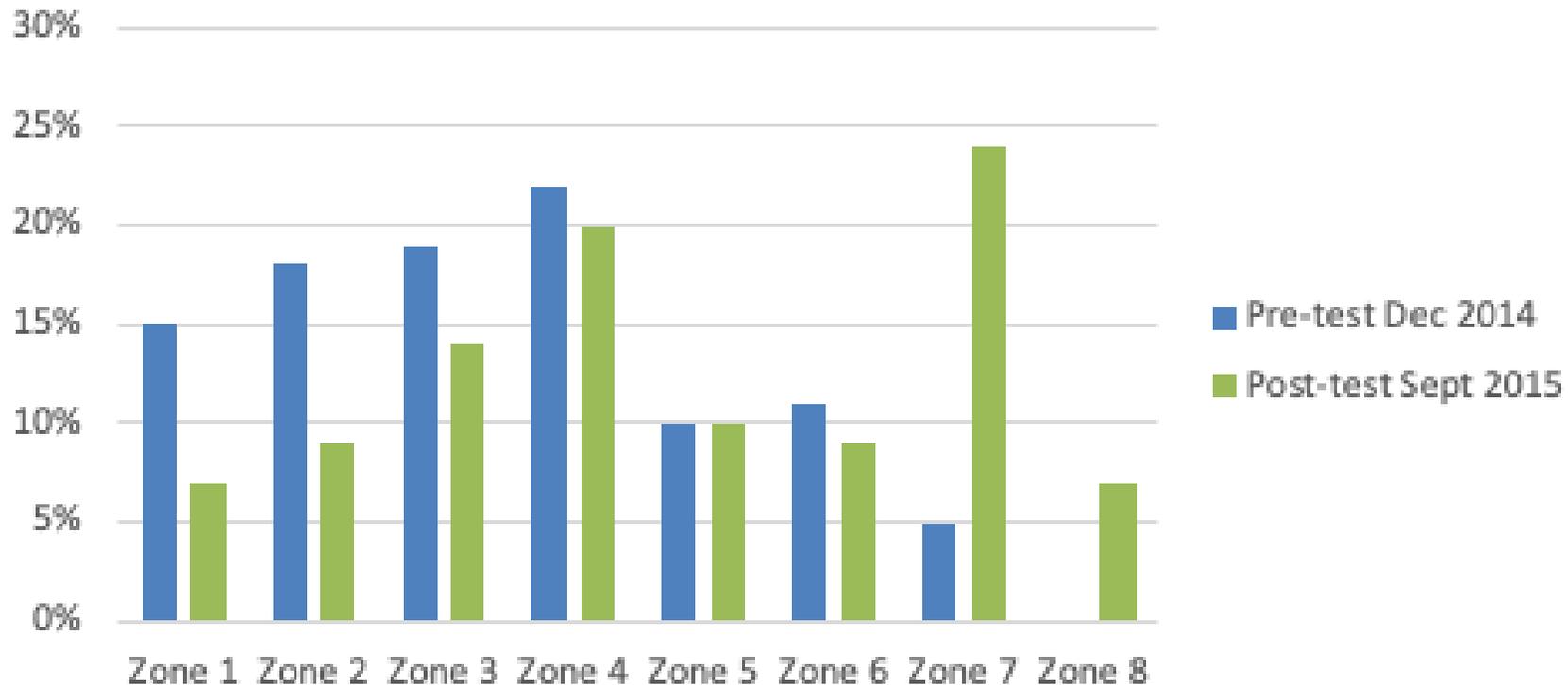
PhD study within the SNMY Project demonstrated major shifts in achievement against the *Learning and Assessment Framework for Multiplicative Thinking* (LAF) as a result of an 18 week, 2 sessions per week teaching program (Breed, 2011).

Participants: 9 Year 6 students identified at Zone 1 of the LAF in May 2004

Results: 8 students achieved at Zone 5 of the LAF in November 2005 (1 top of Zone 4)

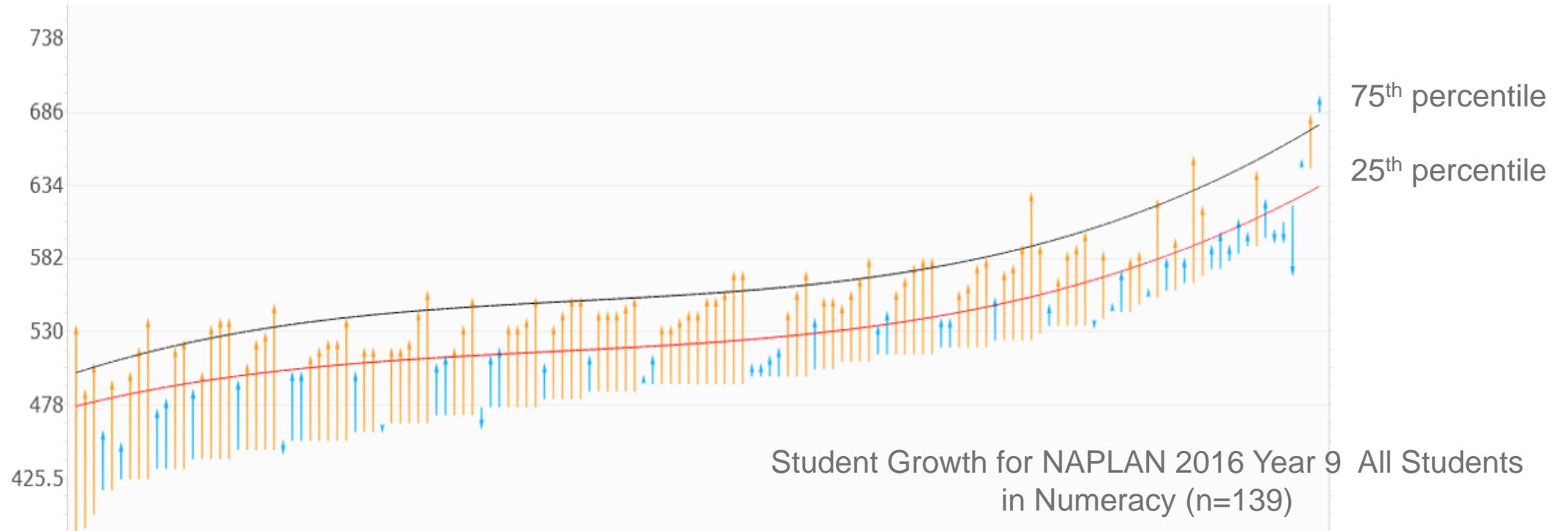
Breed, M. (2011). *Constructing paths to multiplicative thinking*, unpublished PhD Thesis, RMIT University

# Results from an RMFII school who used the SNMY resources and the *Learning Assessment Framework for Multiplicative Thinking* to inform a targeted teaching approach in Year 8 in 2015



*Percentage of students in each Zone of the SNMY LAF in August 2014 (n = 141) and September 2015 (n = 152) (Prasad, November 2016)*

The Year 9 NAPLAN results for the same school in 2016 suggest that the growth in learning was sustained and transferable



Growth in student learning was demonstrated for all bar 5 students and 65% of students grew by more than the expected growth - well above State-wide expectations

# The SNMY Resources

**Instructions**

**Assessment Option Booklet**

**Assessment Materials Multiplicative Thinking Assessment Task Booklet Option 1**

**Scoring Rubrics**

**Scaffolding Numeracy in the Middle Years Linkage Project 2003-2006**

**Scaffolding Numeracy in the Middle Years Linkage Project 2003-2006 LAF Raw Score Translator Option 1**

**Raw Score Translator**

**Learning Progression**

**Teaching Advice**

**INSTRUCTIONS FOR TEACHERS**

Please read the following instructions before administering the Assessment Tasks for Multiplicative Thinking. These tasks are designed to be used with Middle Years Students in Year 4 through to Year 8.

**ADMINISTERING THE TASKS:**

Treat this as you would a normal class assessment activity. If appropriate avoid using the word 'test' and stress that this is about finding out what students know and can do to inform future teaching decisions.

All tasks are contained in the **Assessment Task Booklet (Option 1 or Option 2)** for **Multiplicative Thinking**. The administration of the assessment tasks should be used to locate a student on the Learning and Assessment Framework for Multiplicative Thinking (LAF).

Booklets need to be issued, collected and re-issued until all tasks are completed. Tasks need to be completed in the order they appear in the Task Booklet. The Task Booklet comprises of 1 Extended Task and 4 to 5 Supplementary or Short Tasks.

One full session (at least 40 minutes) should be allocated for the Extended Task. Each Supplementary Task is designed to be completed in about 10 to 15 minutes. While teachers may choose to do more than one Supplementary Task per session, it is suggested that no more than 2 tasks be attempted in any one session unless the session is more than 1 hour long.

Before administering the tasks, students should be given an opportunity to discuss what is expected (see Intervention and Support below).

**MATERIALS:**

Students need their own copy of the Assessment Tasks and whatever materials are specified for the particular task (see Specific Task Advice below). Please read this advice prior to administering the tasks.

Students are expected record all of their work on their copy of the Assessment Tasks so there is no need for scrap paper or jotters etc. If extra space is needed, students should use the back of the page.

**SCAFFOLDING NUMERACY IN THE MIDDLE YEARS LINKAGE PROJECT 2003-2006**

**INSTRUCTIONS FOR TEACHERS**

Please read the following instructions before administering the Assessment Tasks for Multiplicative Thinking. These tasks are designed to be used with Middle Years Students in Year 4 through to Year 8.

**ADMINISTERING THE TASKS:**

Treat this as you would a normal class assessment activity. If appropriate avoid using the word 'test' and stress that this is about finding out what students know and can do to inform future teaching decisions.

All tasks are contained in the **Assessment Task Booklet (Option 1 or Option 2)** for **Multiplicative Thinking**. The administration of the assessment tasks should be used to locate a student on the Learning and Assessment Framework for Multiplicative Thinking (LAF).

Booklets need to be issued, collected and re-issued until all tasks are completed. Tasks need to be completed in the order they appear in the Task Booklet. The Task Booklet comprises of 1 Extended Task and 4 to 5 Supplementary or Short Tasks.

One full session (at least 40 minutes) should be allocated for the Extended Task. Each Supplementary Task is designed to be completed in about 10 to 15 minutes. While teachers may choose to do more than one Supplementary Task per session, it is suggested that no more than 2 tasks be attempted in any one session unless the session is more than 1 hour long.

Before administering the tasks, students should be given an opportunity to discuss what is expected (see Intervention and Support below).

**MATERIALS:**

Students need their own copy of the Assessment Tasks and whatever materials are specified for the particular task (see Specific Task Advice below). Please read this advice prior to administering the tasks.

Students are expected record all of their work on their copy of the Assessment Tasks so there is no need for scrap paper or jotters etc. If extra space is needed, students should use the back of the page.

**SCAFFOLDING NUMERACY IN THE MIDDLE YEARS LINKAGE PROJECT 2003-2006**

**SCORING RUBRIC OPTION 1**

**TABLES AND CHAIRS...**

TASK:	RESPONSE:	SCORE
a.	Incorrect	0
	Correct (8)	1
b.	Incorrect	0
	Correct (10)	1
c.	Incorrect	0
	Correct (3)	1
d.	Incorrect	0
	Correct (5)	1
e.	Incorrect	0
	Correct (9)	1
f.	No response or incorrect	0
	Correct (all gaps filled correctly)	1
g.	No response or incorrect	0
	Results shown as a list or set of ordered pairs, or described additively, eg. "It goes up by 2 each time"	1
	Results shown as a graph or expressed as a rule in words and/or symbols, eg. "The number of people is double the number of tables plus 2 more"	2
h.	No response or incorrect	0
	Incorrect but working and/or explanation indicates attempted use of general rule, or correct (200 people) with no working and/or explanation or evidence of an additive approach, eg. all tables drawn or all ordered pairs listed	1
	Correct (200 people), working and/or explanation indicates general rule recognized in some way, eg. "I doubled it and added 2"	2
i.	No response or incorrect (eg. does not show 6 people and/or table not rectangular)	0
	Correct, ie. rectangular table and either 2 people on longer sides and 1 on each end (arrangement A), or 3 on each of the longer sides (arrangement B)	1

**SCAFFOLDING NUMERACY IN THE MIDDLE YEARS LINKAGE PROJECT 2003-2006**

**Learning Assessment Framework for Multiplicative Thinking**

**Zone 1 - Multiplicative Thinking**

Can solve simple multiplication and division problems involving relatively small whole numbers (e.g., Butterfly House sale and 88) but tends to rely on drawing, models and count-all strategies (e.g., draw and count all girls for part a of Packing Plan). May use skip-counting (repeated addition) for groups less than 5 (e.g., to find number of tables needed to seat up to 20 people in Tables and Chairs).

Can make simple observations from data given in a task (e.g., Adventure Camp a) and reproduce a simple pattern (e.g., Tables and Chairs a to e).

Multiplicative thinking (MT) not fully apparent as no indication that groups are generated as composite units, dealt with systematically, or that the number of groups can be manipulated to support a more efficient calculation.

**Teaching Implications**

**Confidence-building:**

Tracking the count for numbers to 10 (e.g., for 6 this involves working with mental objects for 6 without having to model and/or count). Use flash cards to develop **subitizing** (i.e. ability to say how many without counting) for numbers to 5 initially and then to 10 and beyond using **part-part-whole knowledge** (e.g., 8 is 4 and 4, or 5 and 3 more, or 2 less than 10). Practice regularly.

**Simple skip counting:** to determine how many in a collection and to establish numbers up to 5 as countable objects, eg. count by twos, fives and tens, using concrete materials and a 0-99 Number Chart.

**Mental strategies for addition and subtraction facts to 20 that fit:** Count on from larger (e.g., for 2 and 7, think, 7, 8, 9). Double and near doubles (e.g., use 10+ones and a 2-ten base-10s to show that 7 and 7 is 10 and 4 more, 14), and **Make-Ten** (e.g., for 6 and 8, think, 6, 10, 14, scaffold using open number lines). Double and near mental strategies to solve subtraction problems such as 7 take 2, 12 take 5, and 16 take 9. Practice (e.g., by using Number Charts from Maths 300).

**2-digit place-value - working flexibly with ones and tens:** (by making, naming, recording, comparing, ordering, counting forwards and backwards in place-value parts, and renaming, see Siemon et al., 2016). Play the 'Place-Value Game' (available on the DECSD website).

**Efficient strategies:**

**Doubling (and halving) strategies for 2-digit numbers that do not require renaming** (e.g., 36 and 34, half of 40), add to numbers that require some additional thinking (e.g., to double 35, double 3 tens, double 6 ones, 60 and 12 ones, 72).

**Extended mental strategies for addition and subtraction:** use efficient, place-value based strategies (e.g., 37 and 24, think, 37, 47, 57, 60, 61). Use open number lines to scaffold thinking.

**Efficient and reliable strategies for counting large collections** (e.g., count a collection of 50 or more by 2s, 5s or 10s) with a focus on how to organize the number of groups to facilitate the count (e.g., by arranging the groups systematically in lines or arrays and then skip counting).

**How to make, name and use arrays/groups** to solve simple multiplication or sharing problems using concrete materials, and skip counting (e.g., 1 four, 2 fours, 3 fours ...), leading to more efficient counting strategies based on arrays and the number of rows (e.g., 4 rows of anything, that is, 4 ones, 4 tens, 4 twenties, 4 fours, ...).

**3-digit place-value - working flexibly with tens and hundreds** (by making with base-10, naming, recording, comparing, ordering, counting forwards and backwards in place-value parts, and renaming - see Siemon et al., 2016).

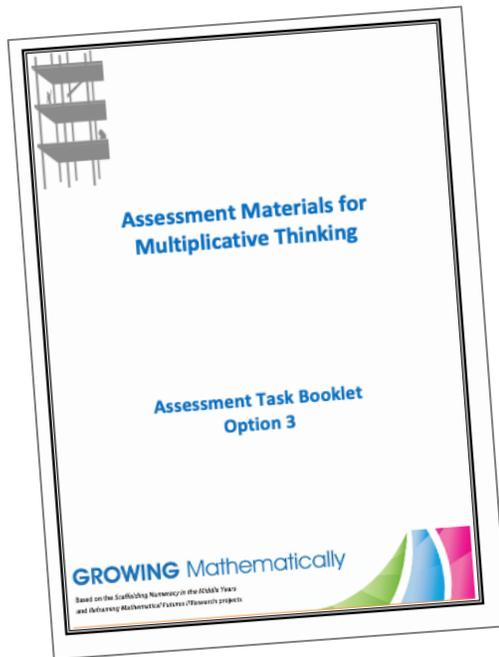
**Strategies for 100** (e.g., test and re-test asking "What do we explore differences in chains in each row, many... all from... shared among 8 as each, how much to...")

**How to explain an** through words and...

<https://www.education.vic.gov.au/school/teachers/teachingresources/discipline/maths/assessment/Pages/scaffoldnum.aspx>

# Two new assessment options

## Option Booklets



**ASSESSING MULTIPLICATIVE THINKING**

**ASSESSMENT TASK BOOKLET FOR MULTIPLICATIVE THINKING  
OPTION 3**

NAME \_\_\_\_\_

YEAR LEVEL: \_\_\_\_\_

This booklet contains an Extended Task and 5 Supplementary or Short Tasks.

**X1 – Trains**  
**S1 – Adventure Camp**  
**S2 – Stained Glass Windows**  
**S3 – Relations**  
**S4 – Skin Rash**  
**S5 – Enlarging Nets**

**INSTRUCTIONS:**

- Please do as much of each task as you can. Some tasks you will find more difficult.
- All working must be shown in this booklet. If you need more space, use the previous page or another space, but make sure we know where to find your work.
- When you are asked to show all your working and explain you do your best, all your working so we can understand your thinking do your best to explain why, in the space provided.
- Don't rub out any work that you think is incorrect. Simply draw a line through it.
- If you have any questions please ask your teacher.

**ASSESSING MULTIPLICATIVE THINKING**

**SCORING RUBRIC OPTION 3**

**TRAINS (ATRNS1)**

SCORE	DESCRIPTION
0	No response or irrelevant response
1	At least two entries correct
2	Table completed correctly (20,26,32,38)

**TRAINS (ATRNS2)**

SCORE	DESCRIPTION
0	No response or irrelevant response
1	Correct response (92) with no explanation/working or incorrect response with working to show some understanding of pattern or incorrect with working to show minor calculation error
2	Correct response with an explanation that reflects the use of an additive strategy (e.g., goes up by 6 or continues table to a train size of 15)
3	Correct response with an explanation of a multiplicative approach expressed in words or as a rule but not in simplest form (e.g., you multiply 6 by 14 and add 8 or working to show $6 \times 14 + 8$ )
4	Correct response with an explanation of a multiplicative approach expressed in words or as a rule in simplest form that recognises the 6 wheels in the engine (e.g., you need to <u>times</u> 15 by 6 and add 2 or $15 \times 6 + 2$ )

## Scoring Rubrics

**TRAINS (ATRNS3)**

SCORE	DESCRIPTION
0	No response or irrelevant response
1	Correct response (No) but with no explanation OR incorrect response with sound reasoning
2	Correct response with reasoning to support conclusion (e.g., $60 - 8$ is 52 and 52 is not divisible by 6 or a size 9 train would have 56 wheels and a size 10 train would have 62 wheels so you can't have a train with 60 wheels.

**RELATIONS (AREL1)**

SCORE	DESCRIPTION
0	No response or irrelevant response
1	General statement (e.g., it goes up by 6) OR incorrect but some evidence that multiplication involved, may or may not recognise addition

**ASSESSING MULTIPLICATIVE THINKING**

**STUDENT SCORE SHEET OPTION 3**

Student Name: \_\_\_\_\_ Year Level: \_\_\_\_\_

Task	Item	Score	Comments
Trains	ATRNS1		
	ATRNS2		
	ATRNS3		
	ATRNS4		
	ATRNS5		
Adventure Camp	adca		
	adcb		
Stained Glass Windows	swga		
	swgb		
	swgc		
Relations	AREL1		
	AREL2		
	AREL3		
Skin Rash	SRASH		
	GENLGD		
Enlarging Nets	GENGLa		
	GENGLb		
	GENGLc		
	GENGLd		
Raw Score			
Percentage			

## Raw Score Translator showing revised learning progression for MT

## Student Response Sheet

**ASSESSING MULTIPLICATIVE THINKING**

**LAF Raw Score Translator Option 3**

The following table is provided to enable teachers to locate students in terms of the Learning and Assessment Framework for Multiplicative Thinking 2021 (LAF) on the basis of their performance on the Assessment Tasks for Option 3 (blue font indicates revisions to the original LAF). To use the table you will need to determine each student's total score by adding the rubric scores assigned to each item (there are 20 items altogether).

Total Score	LAF Zone	Level Description
47-52	8	Can use appropriate representations, language, and symbols to solve and justify a wide range of problems involving unfamiliar multiplicative situations including fractions and decimals. Can justify partitioning. Can use and formally describe patterns in terms of general rules. Beginning to work more systematically with complex, open-ended problems. Can express more complex multiplicative relationships in words or symbols in simplest form and work with two variables simultaneously and equivalent expressions. Recognises and uses scale appropriately and can use a generalised solution strategy in a new context. Beginning to recognise the relationships between perimeter, area, and volume.
35-46	7	able to solve and explain one-step problems involving multiplication and division with whole numbers using informal strategies and/or formal recording. Can solve and explain solutions to problems involving simple patterns, percent, and proportion. May not be able to show working and/or explain strategies for situations involving larger numbers or less familiar problems. Locates fractions using efficient partitioning strategies. Beginning to make connections between problems and solution strategies and how to communicate this mathematically. Able to describe multiplicative relationships as rules in words or symbols but may not express this in simplest form. Can reason algebraically and use symbols to describe what is needed to maintain equivalence in an additive relational context. Can use relationships to calculate simpler volumes and explain thinking in procedural terms.
27-34	6	Can work with the Cartesian Product (for each) idea to systematically list or determine the number of options. Can solve a broader range of multiplication and division problems involving 2-digit numbers, patterns and/or proportion but may not be able to explain or justify solution strategy. Able to rename and compare fractions in the halving family and use partitioning strategies to locate simple fractions. Developing sense of proportion, but unable to explain or justify thinking. Developing a degree of comfort with working mentally with multiplication and division facts. Able to describe and justify rules involving multiplicative relationships Beginning to generalise patterns and formalise rules involving multiplication but may miss more complex patterns involving a constant or ratio or scales requiring estimation or use of diagonals.
23-26	5	Systematically solves simple proportion and array problems suggesting multiplicative thinking. May use additive thinking to solve simple proportion problems involving fractions. Able to solve simple, 2-step problems using a recognised rule/relationship but finds this difficult for larger numbers. Able to order numbers involving tens, ones, tenths and hundredths in supportive

From Growing Mathematically: Multiplicative Thinking. A joint initiative of AAMT and RMFII Project team. Resources available from: <http://www.mathseducation.org.au>

# An updated LAF

8 differentiated levels  
(Zones) of multiplicative  
thinking

Teaching advice to  
consolidate and establish

Teaching advice to  
introduce and develop

<p><b>Zone 3: Sensing</b></p> <p>Demonstrates intuitive sense of proportion (e.g., partial solution to <i>Butterfly House f</i>, correct response to <i>Lemonade Recipe 2</i>) and partitioning (e.g., <i>Missing Numbers b</i>).</p> <p>Works with 'useful' numbers such as 2 and 5, and strategies such as doubling and halving (e.g., <i>Packing Pots b</i>, and <i>Pizza Party c</i>).</p> <p>May list all options in a simple Cartesian product situation (e.g., <i>Canteen Copers b</i>), but cannot explain or justify solutions.</p> <p>Uses abbreviated methods for counting groups such as doubling and doubling again to find 4 groups of repeated halving to compare simple fractions (e.g., <i>Pizza Party c</i>).</p> <p>Beginning to work with larger whole numbers but tends to rely on count all methods or additive thinking to solve problems (e.g., <i>Stained Glass Windows a</i> and <i>b</i>, <i>Tiles, Tiles, Tiles b</i>).</p> <p>Can maintain equivalence across the equals sign (e.g., <i>Relations 1</i>) and extend patterns but may not be able to explain (e.g., <i>Trains 2</i>) or explanation relies on additive thinking (e.g., <i>Board Room Tables 4</i>).</p> <p>Beginning to recognise the importance of scale (e.g., <i>Park Map A1</i>).</p>	<p><b>Consolidate/establish:</b></p> <p><b>Ideas and strategies</b> introduced/developed in the previous Zone</p> <p><b>Introduce/develop:</b></p> <p><b>Place value based strategies</b> for informally solving problems involving single-digit by two-digit multiplication (e.g., for 3 twenty-eights, THINK, 3 by 2 tens, 60 and 24 more, 84) mentally or in writing.</p> <p>Initial recording to support place value for multiplication facts (see Siemon et al, 2021 and <i>There's More to Counting Than Meets the Eye</i>).</p> <p>More efficient strategies for solving number problems involving simple proportion (e.g., recognise as two-step problems, What do I do first? Find value for common amount. What do I do next? Determine multiplier/factor and apply. Why?).</p> <p>How to rename number of groups (e.g., think of 6 fours as 5 fours and 1 more four), Practice (e.g., by using 'Multiplication Toss Game'). Re-name composite numbers in terms of equal groups (e.g., 18 is 2 nines, 9 twos, 3 sixes, 6 threes).</p> <p>Cartesian product or for each idea using concrete materials and relatively simple problems such as 3 tops and 2 bottoms, how many outfits, or how many different types of pizzas given choice of small, large, medium and 4 varieties? Discuss how to recognise problems of this type and how to keep track of the count such as draw all options, make a list or a table (tree diagrams appear to be too difficult at this level, these are included in Zone 5).</p> <p>How to interpret problem situations and solutions relevant to context (e.g., Ask, What operation is needed? Why? What does it mean in terms of original question?). Simple, practical division problems that require the interpretation of remainders relevant to context.</p> <p>Practical sharing situations that introduce names for simple fractional parts beyond the halving family (e.g., thirds for 3 equal parts/shares, sixths for 6 equal parts etc) and help build a sense of fractional parts (e.g., 3 sixths is the same as a half or 50%, 7 eighths is nearly 1, "2 and 1 tenth" is close to 2). Use a range of continuous and discrete fraction models including mixed fraction models.</p> <p>Thirding and fifthing partitioning strategies (see Siemon et al, 2021) through paper folding (kinder squares and streamers), cutting plasticine 'cakes' and 'pizzas', sharing collections equally (counters, cards etc), apply thinking involved to help children create their own fraction diagrams (regions) and number line representations (see Siemon et al 2021). Focus on making and naming parts in the thirding and fifthing families (e.g., 5 parts, fifths) including mixed fractions (e.g., "2 and 5 sixths") and informal recording (e.g., 4 fifths), no symbols. Revisit key fraction generalisations (see Level 2), include whole to part models (e.g., partition to show 3 quarters) and part to whole (e.g., if this is 1 third, show me the whole) and use diagrams and representations to rename related fractions.</p> <p>Extend partitioning strategies to construct number line representations. Use multiple fraction representations.</p> <p>Key fraction generalisations – equal parts, as the number of parts increase the size of the part gets smaller; the number of parts names the part (e.g., 8 parts, eighths) and the size of the part depends upon the size of the whole.</p> <p>Recognise and work with more complex number patterns – explore relational thinking situations when using different operations and use missing numbers on both sides to generate generalisations. (e.g., <i>Maths300: Antimagnons, nRich: Super Shapes</i>), model and discuss how to describe, explain, and generalise simple patterns and relations.</p> <p>Use simple scales (e.g., 1 grid square length to 10 units)</p>
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The Revised Learning and Assessment  
Framework for Multiplicative Thinking (2021)

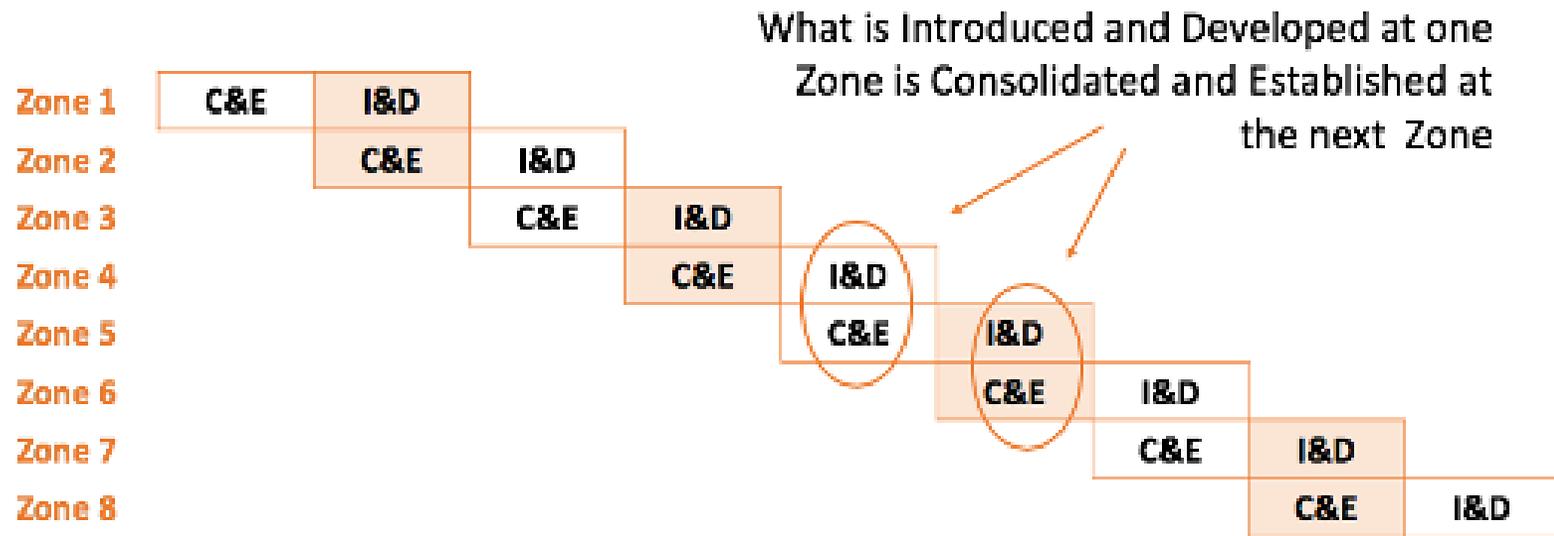
# Implementing a Targeted Teaching Approach

There is no one way to do this - some suggestions:

- Group students across two or more classes in Zone groupings for 1 maths lesson a week, with support from additional staff if possible
- Use a double period once a fortnight to group two or more classes in Zone groupings as above
- Implement an intervention program for 'at risk' students (e.g., staff member works with a small group of three or so students for 20- 25 minutes preferably within the classroom)
- 10-week focus on tasks that afford opportunities to engage with key ideas in mixed ability groups

See Case Studies from Growing Mathematically: Multiplicative Thinking website:  
<http://www.mathseducation.org.au>

# An important note:



Zone 2: Intuitive Modelling		Zone 3: Sensing	
Consolidate & Establish	Introduce & Develop	Consolidate & Establish	Introduce & Develop
<ul style="list-style-type: none"> <li>• Doubling (and halving) strategies</li> <li>• Extended mental strategies for addition and subtraction</li> <li>• Efficient and reliable strategies for counting large collections</li> <li>• How to make, name and use arrays/regions</li> <li>• 3 digit place-value</li> <li>• Strategies for unpacking and comprehending problem situations</li> <li>• How to explain and justify</li> </ul>	<ul style="list-style-type: none"> <li>• More efficient strategies for counting groups</li> <li>• Array/region-based mental strategies for multiplication facts to 100</li> <li>• Commutativity</li> <li>• Informal division strategies</li> <li>• Extended mental strategies for multiplication</li> <li>• Simple proportion problems</li> <li>• How to recognise and describe simple relationships</li> <li>• Language of fractions</li> <li>• Halving partitioning strategy</li> <li>• Key fraction generalisations</li> </ul>	<ul style="list-style-type: none"> <li>• More efficient strategies for counting groups</li> <li>• Array/region-based mental strategies for multiplication facts</li> <li>• Commutativity</li> <li>• Informal division strategies</li> <li>• Extended mental strategies for multiplication</li> <li>• Simple proportion problems</li> <li>• How to recognise and describe simple relationships</li> <li>• Language of fractions</li> <li>• Halving partitioning strategy</li> <li>• Key fraction generalisations (equal parts)</li> </ul>	<ul style="list-style-type: none"> <li>• Place-value based strategies</li> <li>• Simple proportion problems (intuitive)</li> <li>• Cartesian product or for each idea</li> <li>• How to interpret problem situations and solutions relevant to context</li> <li>• Interpretation of remainders</li> <li>• Practical sharing situations to build a sense of fractional parts,</li> <li>• Thirthing and fifthing partitioning strategies to create fraction models</li> <li>• Extend partitioning strategies</li> <li>• Key fraction generalisations (number of parts in relation to size)</li> </ul>

# 'Threads' can be used to inform task selection

Consolidating & Establishing Zone 1	Introducing & Developing Zone 1/Consolidating & Establishing Zone 2	Introducing & Developing Zone 2
<ul style="list-style-type: none"> <li>• Trusting the count (subitising and part-part-whole knowledge)</li> <li>• Simple skip counting (2s &amp; 5s)</li> <li>• Mental strategies for addition and subtraction facts to 20</li> <li>• 2 digit place-value</li> </ul>	<ul style="list-style-type: none"> <li>• Doubling (and halving) strategies</li> <li>• Extended mental strategies for addition and subtraction</li> <li>• Efficient and reliable strategies for counting large collections</li> <li>• How to make, name and use arrays/regions</li> <li>• 3 digit place-value</li> <li>• Strategies for unpacking and comprehending problem situations</li> <li>• How to explain and justify</li> </ul>	<ul style="list-style-type: none"> <li>• More efficient strategies for counting groups</li> <li>• Array/region-based mental strategies for multiplication facts to 100</li> <li>• Commutativity</li> <li>• Informal division strategies</li> <li>• Extended mental strategies for multiplication</li> <li>• Simple proportion problems</li> <li>• How to recognise and describe simple relationships</li> <li>• Language of fractions</li> <li>• Halving partitioning strategy</li> <li>• Key fraction generalisations</li> </ul>

# The Assessment for Common Misunderstandings (Big Ideas) Resources

## 5.3 Understanding Scale Factors Tool<sup>1</sup>

**Materials:**  
 2 Pentagon Cards (cut out so that they can be manipulated, see Level 5 Resources)  
 Dot Paper Worksheet (see Level 5 Resources)  
 Map Worksheet (see Level 5 Resources) and pen  
 A ruler

**Instructions:**  
 Place the two cards in front of the student and say, "What can you tell me about these two shapes? ... Note student's response, then say, "How would you tell a friend to draw the large shape if you could only show them the small shape?" Note student's response.

Place the Dot Paper Worksheet in front of the student and say, "Could you make this shape half as big please?" Note and retain student's response.

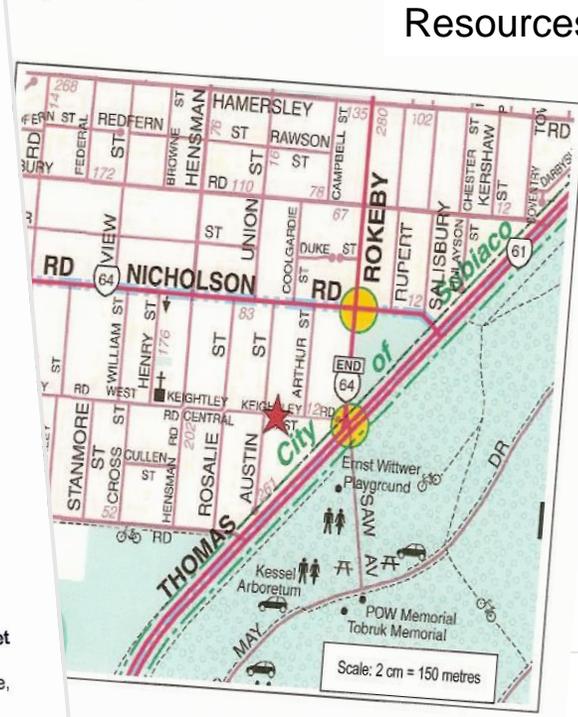
Place the Map Worksheet in front of student and say, "This is a map of a suburb in Perth. Can you find Nicholson Road?" ... Point to the scale and ask, "Can you tell me what this means? ... Can you give me an example?" ...

If no response, say, "If you walked the full length of Arthur Street (indicate this, it is just to the right of the red star), about how far would you have walked?" ... Note student's response, then say, "Jo walks to Rosalie School which is here (indicate the red star). She lives on the corner of Nicholson and Rupert Street (indicate). About how far does she walk to school in the morning?" ... Indicate that the bottom of the page can be used for any working required. Note student's response and ask him/her to explain their reasoning.

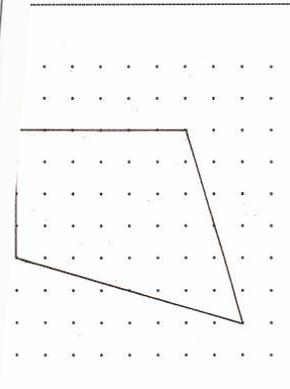
If correct (ie, about 375 metres), say, "Thuan lives on the corner of Redfern Street and View Street (indicate upper left hand corner of the map). About how far does he have to ride to school?" ... Note student's strategy and response in each case, retain any working.

Diagnostic Interview

Map Worksheet:



Resources



Teaching Advice

## 5.3 Understanding Scale Factors Tool

Proportional reasoning is often more apparent in relation to visual images, eg, recognising shapes that have been enlarged or reduced, than it is in word problems that require interpretation relative to context. However, where students have had a limited exposure to the skills and strategies needed to enlarge or reduce shapes and/or to construct and interpret scale drawings there is a distinct possibility that misunderstandings will arise. One of these is the tendency to focus on area when attempting to identify 'how many times larger' one shape is of a smaller, similar shape.

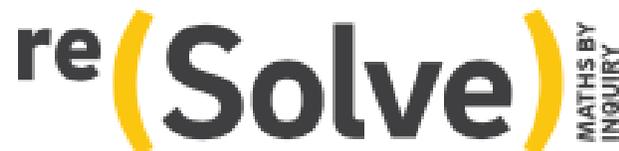
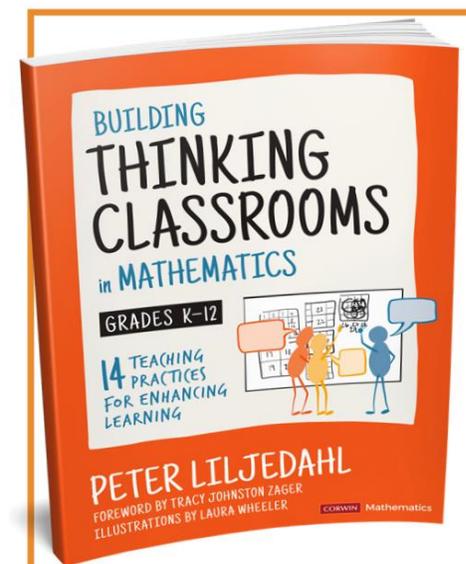
This task examines the extent to which students are able to recognise and describe enlargements, and use a scale factor to reduce a shape and estimate distances on a scale map.

Observed response	Interpretation/Suggested teaching response
Little/no response, possibly recognises shapes and/or notes one is bigger than the other, may be able to explain meaning of map scale	<p>May not understand the spatial task or have access to the skills and strategies needed to interpret maps</p> <ul style="list-style-type: none"> <li>Ensure that what is meant by statements such as, "3 times as big as" or "half the size of" are understood, ie, they can be modelled and interpreted relative to context</li> <li>Use peg boards, dot paper, cm grid paper etc to enlarge and reduce shapes by simple scalar amounts, discuss this in terms of what happens to corresponding sides (they are multiplied or divided by the same factor)</li> <li>Discuss which attribute is relevant and why for 2 D shapes (ie, length not area, as object is to produce similar shapes)</li> <li>Provide opportunities to work with maps and scale diagrams, make thinking explicit, scaffold appropriate strategies for calculating or estimating distances</li> </ul>
Recognises shapes are the same, may identify scale factor (3) but unable to halve the quadrilateral, although may make a start (eg, draw relevant diagonal), explains meaning of map scale but may not be able to use this to provide an example or reasonable estimates for both map questions (eg, may treat as 1 cm is 350 metres)	<p>Suggests a limited understanding of the scale factor idea for multiplication,</p> <ul style="list-style-type: none"> <li>Use cm grid/dot paper, peg boards etc to review the processes and language involved in enlarging and reducing 2D shapes by a range of different factors starting with simple shapes such as rectangles and moving to more complex shapes such as scalene triangles and irregular quadrilaterals</li> <li>Practice map reading skills and strategies, talk about the use of scales, construct scale drawings of the classroom, school grounds, students homes and/or backyards etc. Discuss equivalent scales (eg, 2 cm to 150 metres is the same as 1 cm to 75 metres)</li> <li>Explicitly link the use of scales to multiplication using the term <i>scale factor</i>, explore the impact of different scale factors, including scale factors less than 1</li> </ul>
Recognises scale factor for pentagons, able to halve the quadrilateral, may use diagonal from right angle vertex to opposite vertex to locate corresponding point, can explain meaning of map scale and provide reasonable estimates of distances	<p>Indicates a solid understanding scale factors and how it relates to multiplication in this context</p> <ul style="list-style-type: none"> <li>Provide opportunities for students to work with an extended range of scale factors, eg, very large whole numbers, decimals, mixed fractions, percentages, ratios, etc</li> <li>Extend scale drawing skills and strategies to include the idea of perspective and the use of a centre of enlargement (or dilation)</li> <li>Link solution strategies to proportional reasoning problems more generally, eg, finding for 1 and multiplying or finding for a common composite unit and multiplying</li> <li>Practice map reading skills and strategies, talk about the use of scales</li> </ul>

<https://www.education.vic.gov.au/school/teachers/teachingresources/discipline/maths/assessment/Page/s/misunderstandings.aspx>

# Targeted Teaching $\neq$ Ability Grouping (Streaming)

- Students need to work in mixed ability groups on rich, accessible, but challenging tasks for the majority of maths time (Boaler, 2008; Sullivan, 2011)
- Flexibility and student choice are key



engaging lessons and professional support



## Ability Grouping



Emeritus Professor Dianne Siemon  
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<https://www.education.vic.gov.au/Documents/school/teachers/teachingresources/discipline/maths/ability-grouping.pdf>