A scar tree on the Bundoora Campus of RMIT University

MULTIPLICATIVE THINKING – WHAT IS IT AND WHY IS IT IMPORTANT?

A Keynote Address to the Bringing Maths to Life Conference, UNSW, November 2022

By Emeritus Professor Dianne Siemon - RMIT University

I would like to show my respects and acknowledge the Bedegal people who are the Traditional Custodians of the Land on which this meeting takes place, and to Elders past and present

Multiplicative Thinking – a really BIG IDEA

TAIL STRIKE …

On 20 March 2009, an Emirates Airbus 340-500 with 257 passengers, 12 cabin crew and 4 flight crew safely returned to Melbourne Airport after a tail strike on take-off.

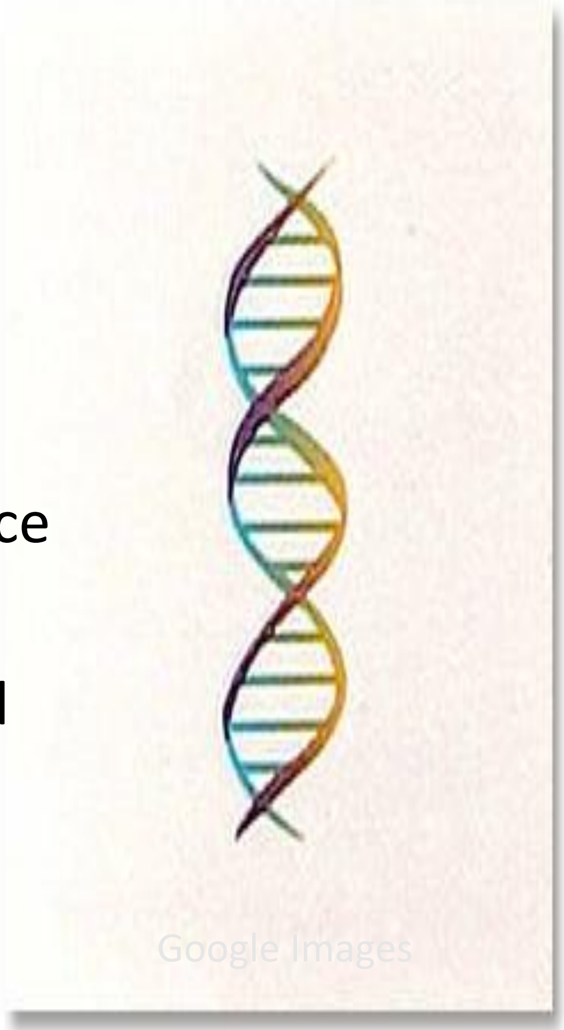
The investigation found that the take-off weight of the aircraft entered into the flight computer was 100 tonnes less than the actual weight …an error of 1 in the hundreds place!

DNA EVIDENCE …

Some years back, the American Jury Association conducted a mock trial to explore some of the issues involved in the use of DNA evidence.

Two juries were convened to hear exactly the same evidence with one difference:

> Jury 1 was told that there was a 2% chance that the DNA evidence found at the crime site did not come from the accused.
>
> Jury 2 was told that the probability that the DNA evidence found at the crime site did not come from the accused was 0.02.
>
> The conviction rates were significantly different!


Google Images

Google Images

MEDICATION …

A seriously ill colleague was taken to hospital and lapsed into a coma.

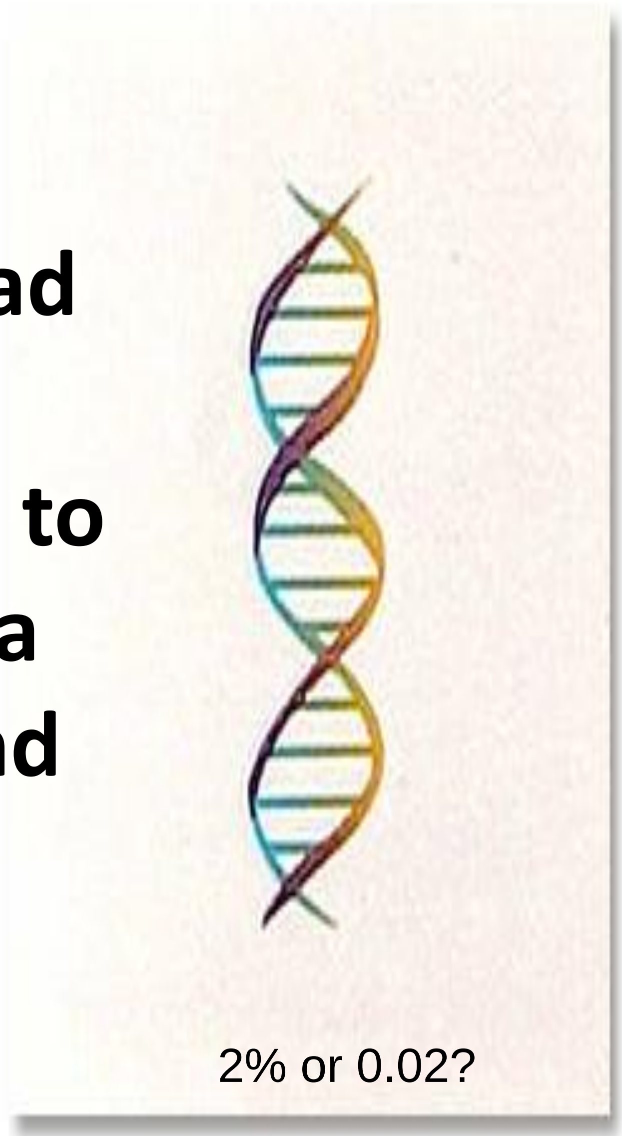
A day or so later he was semi-conscious but unable to move or speak …

He was aware that he was about to be medicated but became concerned when he heard two nurses debating as to whether or not a 1% solution meant 1 mL in 10 mL or 1 mL in 100 mL …

Actual: 362.9 tonnes
Entered: 262,9 tonnes

1%?
1 in 10 or
1 in 100?

The ability to read and interpret numbers relative to context can be a matter of life and death!



2% or 0.02?

Taught but not learnt?

Year 5: Compare, order, and represent decimals (ACARA)

Year 6: Make connections between equivalent fractions, decimals, and percentages (ACARA)

Year 8: Solve problems involving the use of percentages … (ACARA)

Reading and interpreting numbers relative to context requires more than discrete skills and competencies! It requires:

- A deep understanding of the base ten system of numeration
- Recognition that fractions, decimals, and percentages are just different ways to represent a multiplicative relationship between two quantities.
- An alertness to context, a critical stance, and an inclination to check for reasonableness

A significant issue

Only 56% of Australian 15-year-olds solved this PISA problem in 2012?

You are making your own dressing for a salad.

Here is a recipe for 100 millilitres (mL) of dressing.

Salad oil: 60 mL

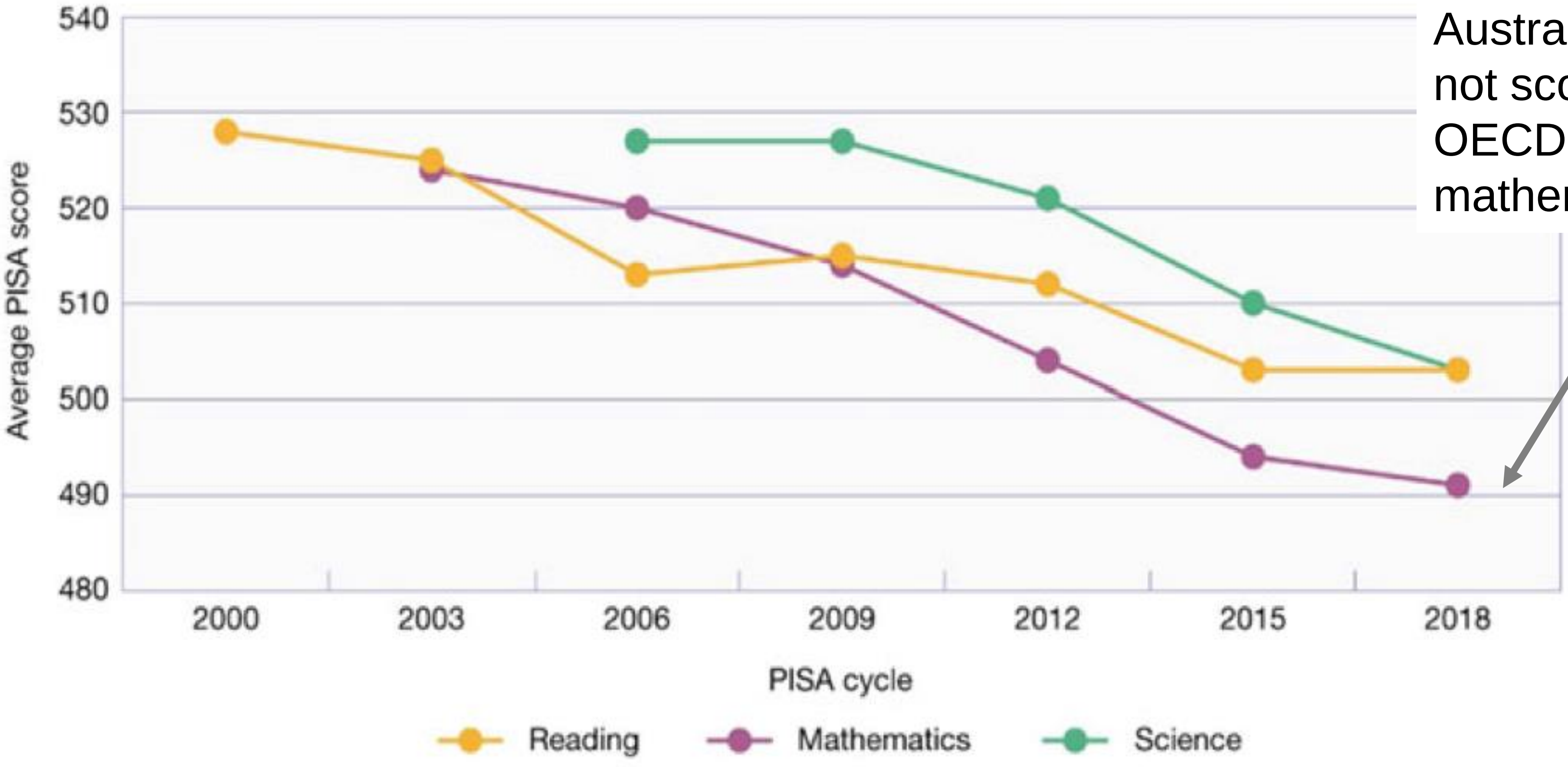
Vinegar: 30 mL

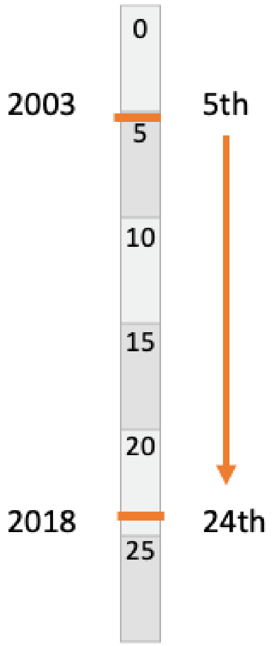
Soy Sauce: 10 mL.

How many millilitres (mL) of salad oil do you need to make 150 mL of this dressing?

What we are doing is not working

2018 is the first time Australian students have not scored above the OECD average in mathematical literacy

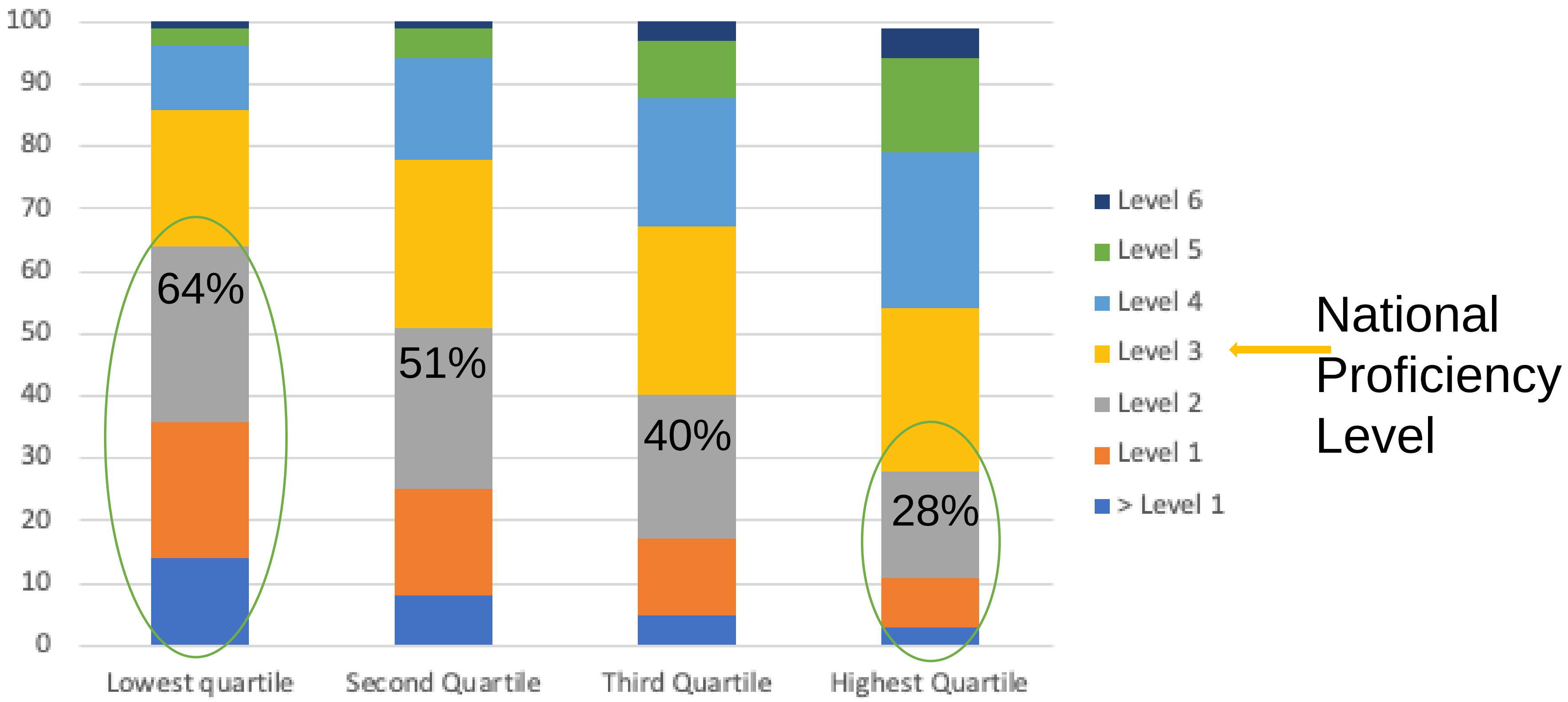




Australian students' performance in PISA 2000-2018

Source: Thomson, S. (2021). *Australia: PISA Australia – Excellence and Equity?* (p. 31). ACER
https://research.acer.edu.au/cgi/viewcontent.cgi?article=1052&context=ozpisa

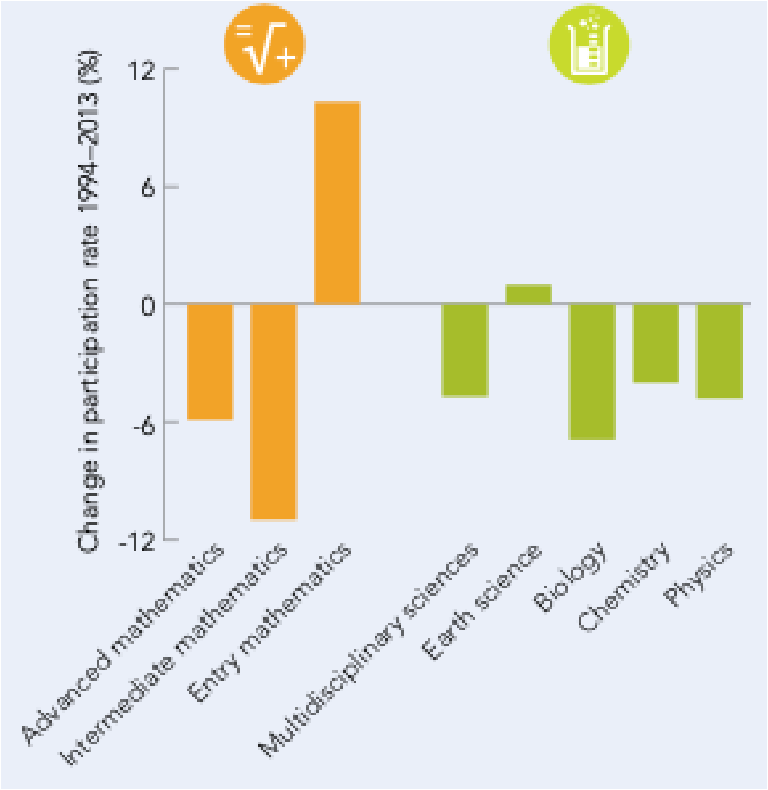
Students from lower socioeconomic backgrounds are more than twice as likely to be below the National Proficiency Level than students from higher socioeconomic backgrounds

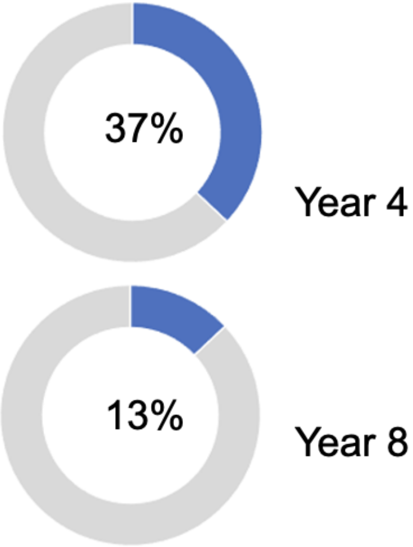


Percentage of students across mathematical proficiency levels by socioeconomic background PISA 2018 (Source: Thomson, 2021)

Participation in most Year 12 **maths** and **science** subjects is declining and for **science** it is the **lowest in 20 years**

Australian Office of the Chief Scientist. (2017). https://www.chiefscientist.gov.au/sites/default/files/2-Science-and-Maths-in-Australian-Secondary-Schools-datasheet-Web.pdf





Students are losing interest in school mathematics at an early age

Thompson et al., 2017. TIMSS2015 – Reporting Australia’s Results, ACER

What can we learn from research?

… *excellence in teaching is the single most powerful influence on student achievement* (Hattie, 2003, p. 4)

… *well-grounded evidence of pupils' progressions in learning is crucial to the work of teachers* (Black, Wilson, & Yeo, 2011, p. 71)

... *assessment for learning, is one of the most powerful ways of improving student achievement* (Wiliam, 2013, p. 15)

Assessment for learning

An assessment functions formatively to the extent that evidence about student achievement is ***elicited, interpreted****, and* ***used*** *by teachers, learners, or their peers to make decisions about the next steps in instruction that are likely to be better, or better founded, than the decisions they would have made in the absence of that evidence.*

(Wiliam, 2011, p. 43, my emphasis)

It’s not rocket science

A social-constructivist view of learning requires that we recognise and act on three key processes in learning. An understanding of:

- where the learner is right now;
- where the learner needs to be; and
- how to get there.

Wiliam, D. (2013). Assessment: The bridge between teaching and learning. *Voices from the Middle, 21*(2), 15 – 20

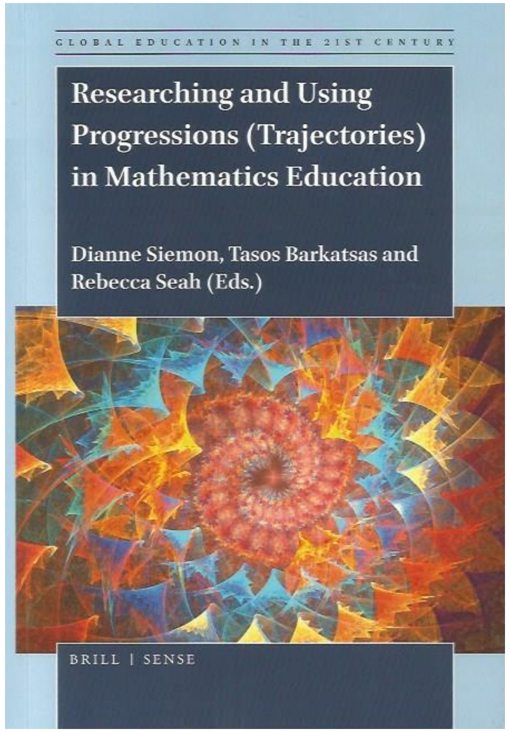
But **where** in relation to what?

Year level curriculum expectations or what research suggests is most likely to make a difference?

Learning Progressions

*In recent years attention has turned to the **development of evidenced-based learning trajectories (or progressions)** as a means of identifying what mathematics is important and how it is understood over time.*

*But for this information to be useful to practitioners, it needs to be accompanied by **accurate forms of assessment** that locate where learners are in their learning journey and **evidenced-based advice** about where to go to next.* (Siemon, 2019, p.6)



Siemon, D. (2019). Knowing and building on what students know: The case of multiplicative thinking. In D. Siemon, T. Barkatsas & R. Seah (Eds.), *Researching and Using Progressions (Trajectories) in Mathematics Education (pp. 6-31).* Brill-Sense

Middle Years Numeracy Research Project (MYNRP, 1999-2001) - explored number sense, measurement & data sense, and spatial sense in in a structured sample of just under 7000 students in Years 5 to 9 students using rich tasks and Rasch modelling

Numeracy

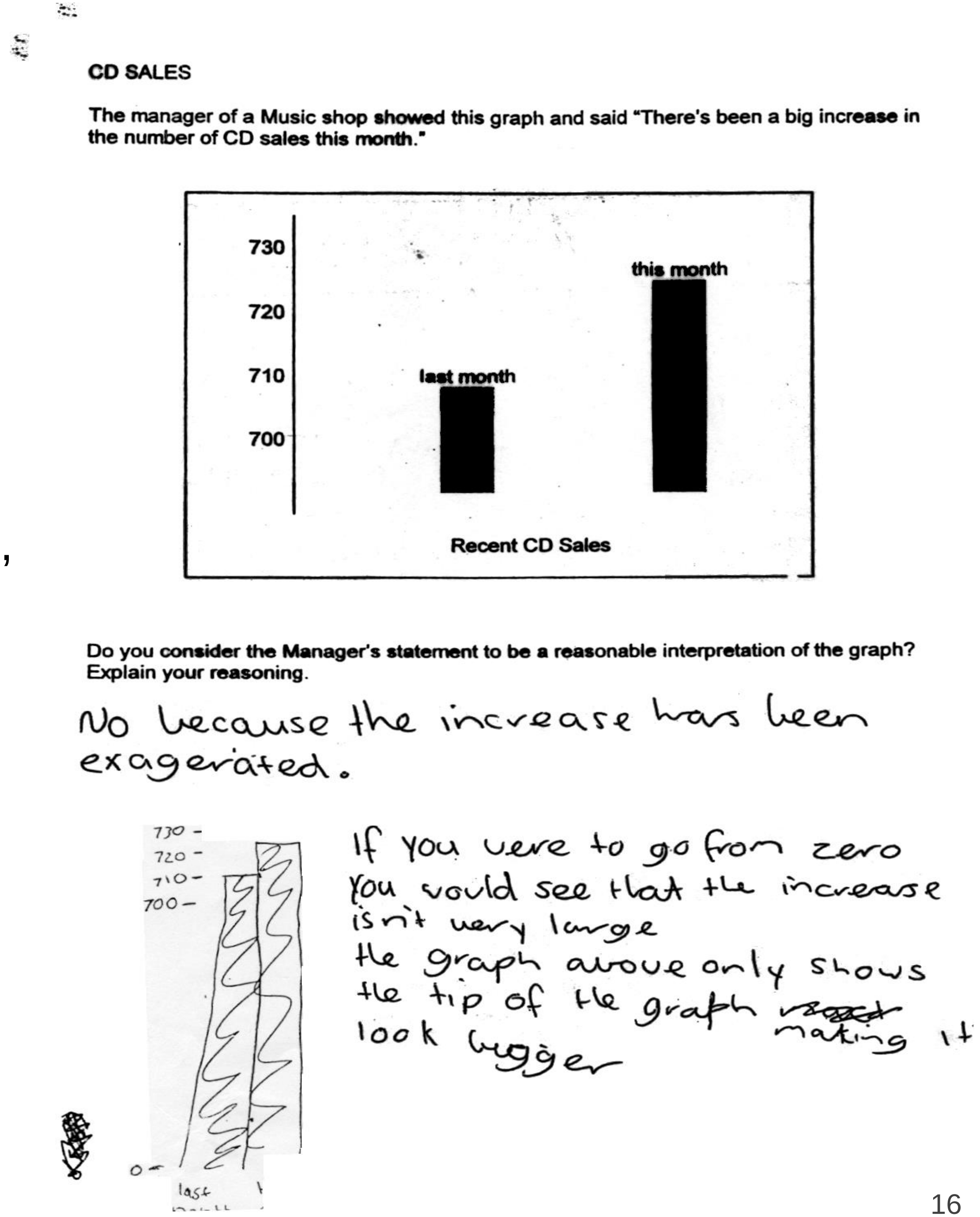
- core mathematical knowledge - in this case, number sense, measurement and data sense and spatial sense as elaborated in the *National Numeracy Benchmarks for Years 5 and 7* (1997);
- the capacity to critically apply what is known in a particular context to achieve a desired purpose; and the
- actual processes and strategies needed to communicate what was done and why.

(Siemon & Virgona, 2001)

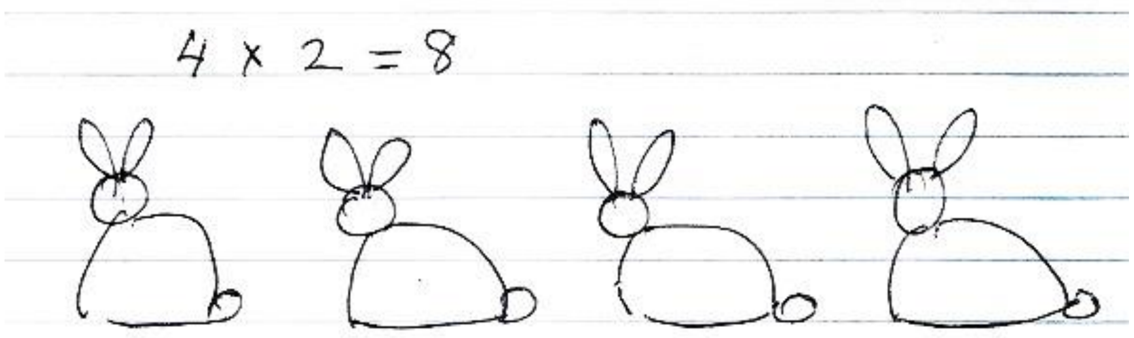
A rich task and scoring rubric*

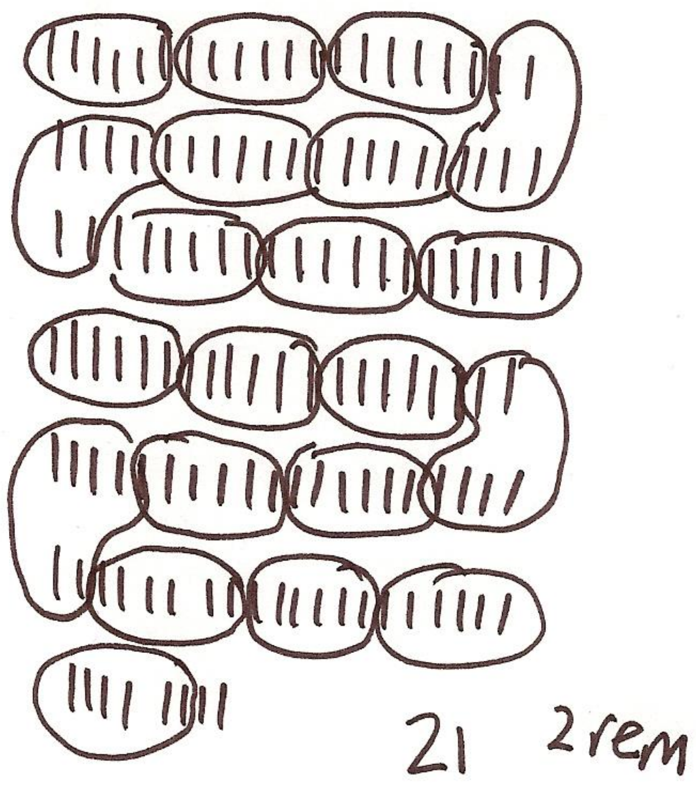
0. No response or ‘yes’ or ‘no’ without a reason.
1. Reasoning based on numbers alone, no recognition that ‘big’ is relative.
2. Reasoning shows some recognition that ‘big’ is relative to total sales, but unsupported conclusion, little/no explanation (e.g., “it depends ...”).
3. Reasoning concludes that increase is not ‘big’ relative to total sales, some attempt to relate this to proportion (e.g., “15 out of 725 is not very big”).
4. Correct conclusion, “not big”, %, fractions, ratio used correctly to support well-reasoned explanation.

* Task adapted from a PISA item, response from MYNRP



A Year 8 student's response to the question: What sort of maths do you really like doing?





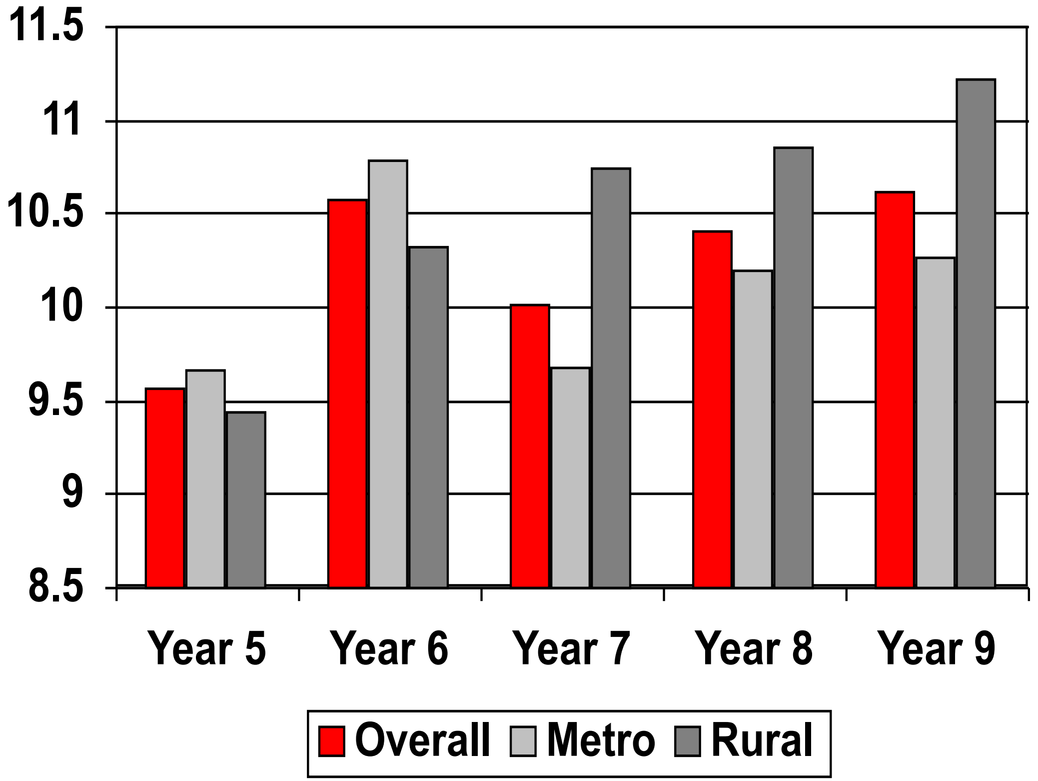
A Year 7 student's response to the problem: 128 ÷ 6 = □

Suggests understanding limited to the **equal groups** idea for multiplication and the corresponding **quotition** idea for division

Two responses from the 'At Risk' Individual Student Interviews, MYNRP (2000)

MYNRP Results

- There is a 5 to 7-year range in student mathematics achievement in each year level from Years 5 to 9
- There is as much variation within schools as between schools (individual teachers make a significant difference to student learning);
- The needs of many students, but particularly those ‘at risk’ or ‘left behind’, were not being met.
- Differences in performance were almost entirely due to an inadequate understanding of fractions, decimals, and proportion (i.e., **multiplicative thinking**), and a reluctance/inability to explain/justify solutions.



Mean Adjusted Logit Scores by Location November 1999 (N=6879)

***Scaffolding Numeracy in the Middle Years* (SNMY) Project (2004-2006)** - explored the development of multiplicative thinking in a purposeful sample of just over 3200 Year 4 to 8 students using rich tasks and Rasch modelling

Multiplicative Thinking defined in terms of:

- a capacity to work flexibly and efficiently with an extended range of numbers (for example, larger whole numbers, decimals, common fractions, ratio, and per cent);
- an ability to recognise and solve a range of problems involving multiplication, division and/or proportional reasoning; and
- the means to represent and communicate this effectively in a variety of ways (e.g., words, diagrams, symbolic expressions, and written algorithms),

(Siemon, Breed, Dole et al, 2006)

Multiplicative thinking is fundamentally different to additive thinking

A fruit stall sold 14 punnets of strawberries. If they had 45 punnets to start with, how many did they have left for sale?

Part	Part
Whole	

Additive Structure

12 strawberry plants per row, 18 rows, how many plants?

Multiplicative Structure

M_1	M_2
1	12
18	?

Multiplicative thinkers:

- are **alert to and reason with relationships between different quantities** (e.g., distance and time, kilograms and minutes)
- recognise and **use the *for each* and *factor-factor-product* ideas** to solve problems,
- no longer rely on *repeated addition* or a count of equal groups.

Because simple multiplicative problems can be solved additively by repeated addition or a learnt algorithm and known facts, **it can be difficult to determine whether or not a student is thinking multiplicatively.**

SNMY Extended Task

BUTTERFLY HOUSE…

Some children visited the Butterfly House at the Zoo.

They learnt that a butterfly is made up of 4 wings, one body and two feelers.
While they were there, they made models and answered some questions.
For each question, explain your working and your answer, in as much detail as possible.

a. How many wings, bodies and feelers would be needed for 7 model butterflies?

______________ wings
______________ bodies
______________ feelers

b. How many complete model butterflies could you make with 16 wings, 4 bodies and 8 feelers?

This task had 9 items altogether including:

Items of **increasing complexity**, eg, "How many complete model butterflies could you make with 29 wings, 8 bodies and 13 feelers?"

Items involving **simple proportion and rate**, eg, "To feed 2 butterflies, the zoo needs 5 drops of nectar per day. How many drops would be needed per day to feed 12 butterflies?" …and

Items involving the **Cartesian product**, eg, given 3 different body colours, 2 types of feelers and 3 different wing colours, "How many different model butterfles could be made?"

Adapted from 'Butterflies and Caterpillars' (Kenney, Lindquist & Heffernan, 2002) for the SNMY Project (2003-2006)

SNMY Short Task

ADVENTURE CAMP ...

Camp Reefton offers 4 activities. Everyone has a go at each activity early in the week. On Thursday afternoon students can choose the activity that they would like to do again. The table shows how many students chose each activity at the Year 5 camp and how many chose each activity at the Year 7 camp a week later.

	Rock Wall	Canoeing	Archery	Ropes Course
Year 5	15	18	24	18
Year 7	19	21	38	22

Camp Reefton Thursday Activities

a. What can you say about the choices of Year 5 and Year 7 students?

b. The Camp Director said that canoeing was more popular with the Year 5 students than the Year 7 students. Do you agree with the Director's statement? **Use as much mathematics as you can to support your answer.**

Open-ended question

Problem solving, solution strategy unclear

Reading and interpreting quantitative data relative to context

Recognising relevance of proportion

Mathematics used (e.g., percent, fractions, ratio)

SNMY Project (2003-2006)

10.

ADVENTURE CAMP ...

Camp Reefton offers 4 activities. Everyone has a go at each activity early in the week. On Thursday afternoon students can choose the activity that they would like to do again.

The table shows how many students chose each activity at the Year 5 camp and how many chose each activity at the Year 7 camp a week later.

	Rock Wall	Canoeing	Archery	Ropes Course
Year 5	15	18	24	18
Year 7	19	21	38	22

Camp Reefton Thursday Activities

a. What can you say about the choices of Year 5 and Year 7 students?

15 19
18 21
24 38
18 22
75 100

The majority of both groups chose Archery while the other activities have around the same ~~no~~ numbers except for the rock wall which more year 7s chose

b. The Camp Director said that canoeing was more popular with the Year 5 students than the Year 7 students. Do you agree with the Director's statement? **Use as much mathematics as you can to support your answer.**

15 | 19
18 | 21
24 | 38
18 | 22
75 | 100

The director is probably right because all together there are more year 7s than 5s so that the percentage of 5s would be higher than 7s

25
75
100

18
6

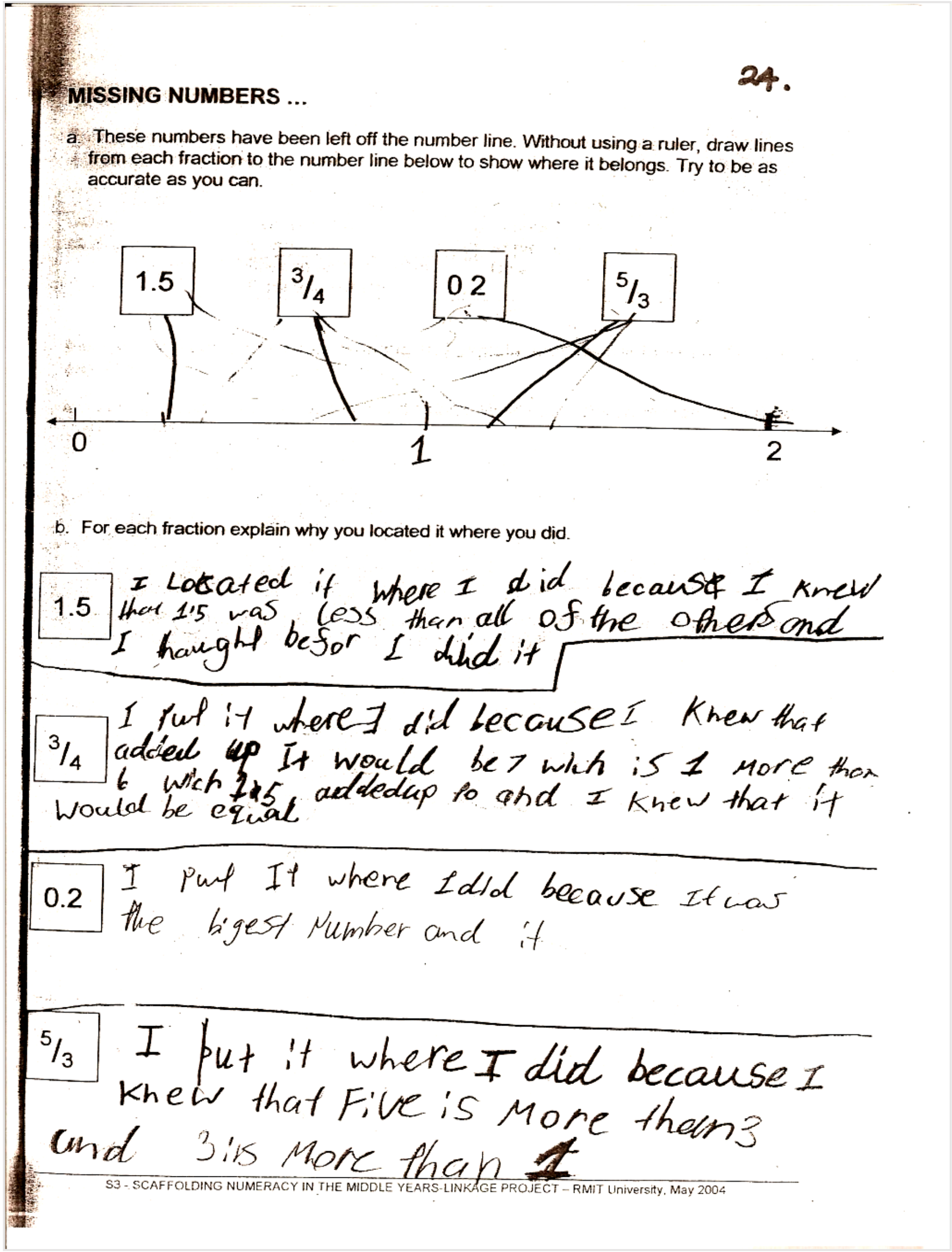
4) 25.020
6.25

YEAR 5 — about 24%

YEAR 7 — 21%

ADVENTURE CAMP …

TASK:	RESPONSE:	SCORE
a.	No response or incorrect or irrelevant statement	**0**
	One or two relatively simple observations based on numbers alone (e.g., “Archery was the most popular activity for both Year 5 and Year 7 students”, “More Year 7 students liked the rock wall than Year 5 students”)	**1**
	At least one observation which recognises the difference in total numbers (e.g., “Although more Year 7s actually chose the ropes course than Year 5, there were less Year 5 students, so it is hard to say”)	**2**
b.	No response	**0**
	Incorrect (No), argument based on numbers alone (e.g., “There were 21 Year 7s and only 18 Year 5s”)	**1**
	Correct (Yes), but little/no working or explanation to support conclusion	**2**
	Correct (Yes), working and/or explanation indicates that numbers need to be considered in relation to respective totals (e.g., “18 out of 75 is more than 21 out of 100”), but no formal use of fractions or percent or further argument to justify conclusion	**3**
	Correct (Yes), working and/or explanation uses comparable fractions or percents to justify conclusion (e.g., “For Year 7 it is 21%. For Year 5s, it is 24% because 18/75 = 6/25 = 24/100 = 24%”)	**4**



24.

MISSING NUMBERS ...

a. These numbers have been left off the number line. Without using a ruler, draw lines from each fraction to the number line below to show where it belongs. Try to be as accurate as you can.

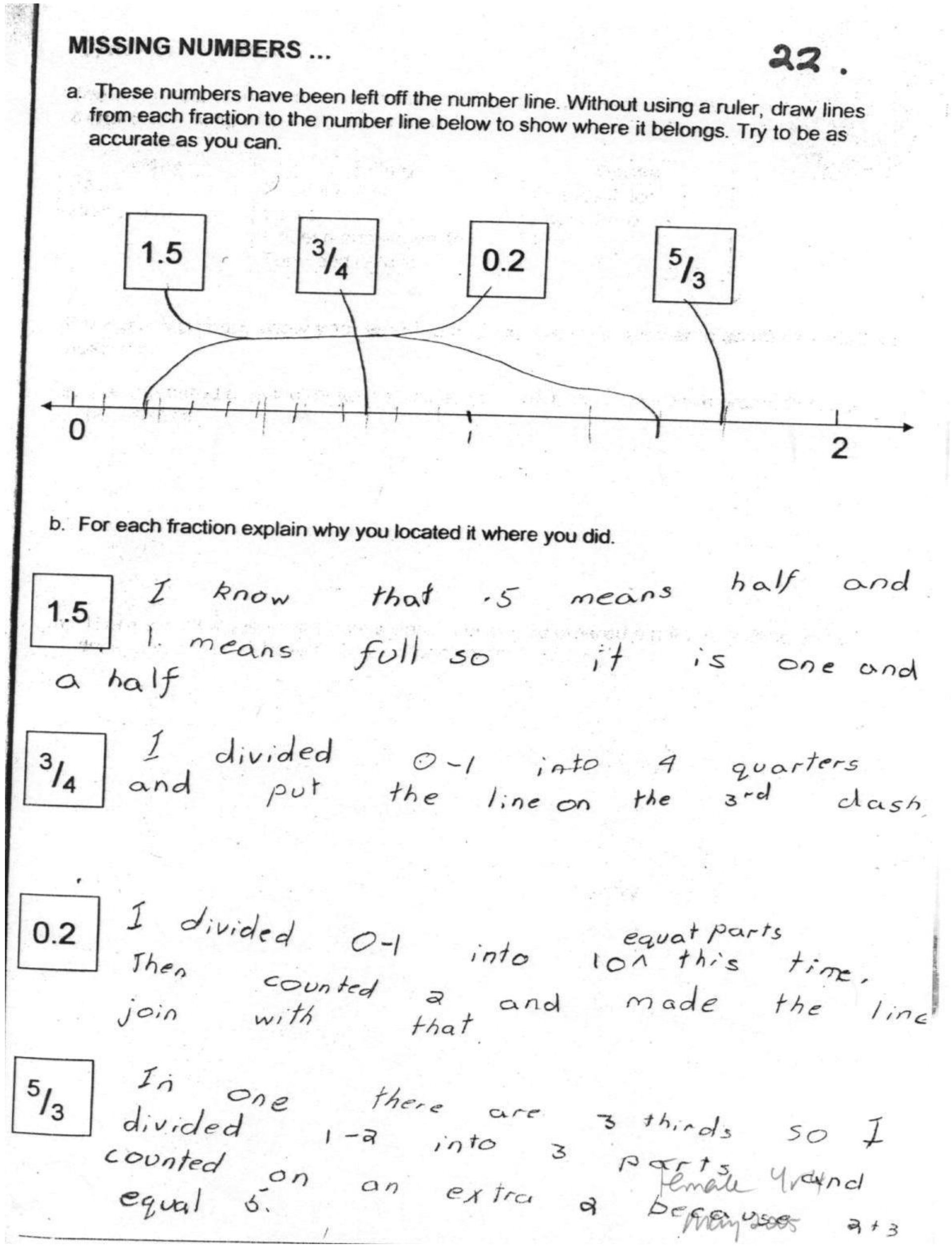
1.5 $^3/_4$ 0 2 $^5/_3$

0   1   2

b. For each fraction explain why you located it where you did.

1.5	I Located it where I did because I Knew that 1.5 was less than all of the others and I thought befor I did it
$^3/_4$	I put it where I did becouse I Knew that added up It would be 7 wich is 1 more than 6 wich 1.5 addedup to and I Knew that it Would be equal
0.2	I put It where I did because It was the bigest Number and it
$^5/_3$	I put it where I did because I Knew that Five is More than 3 and 3 is More than 1

S3 - SCAFFOLDING NUMERACY IN THE MIDDLE YEARS-LINKAGE PROJECT – RMIT University, May 2004



MISSING NUMBERS ...   22.

a. These numbers have been left off the number line. Without using a ruler, draw lines from each fraction to the number line below to show where it belongs. Try to be as accurate as you can.

1.5 $^3/_4$ 0.2 $^5/_3$

0   2

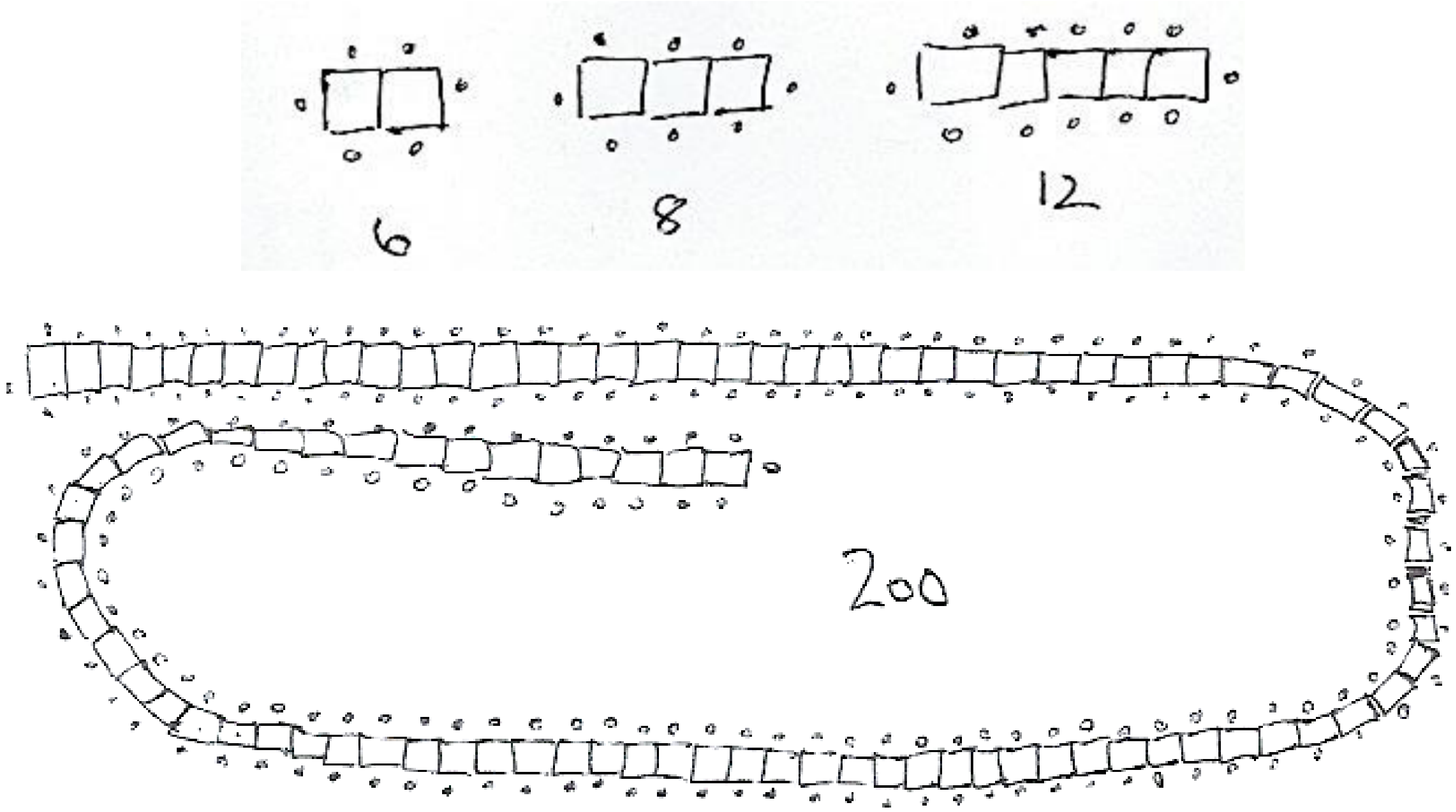
b. For each fraction explain why you located it where you did.

1.5	I know that .5 means half and 1 means full so it is one and a half
$^3/_4$	I divided 0-1 into 4 quarters and put the line on the 3rd dash
0.2	I divided 0-1 into 10 equal parts this time. Then counted 2 and made the line join with that
$^5/_3$	In one there are 3 thirds so I divided 1-2 into 3 parts and counted on an extra 2 because 2+3 equal 5.

Two Year 4 Student Responses to *Missing Numbers* Short Task (SNMY, May 2004)

Naïve Response to Tables and Chairs Extended Task:

20% Year 4 and close to 6% Year 8 students drew a diagram (i.e., made and counted all) to find number of people sitting at 99 tables



Scaffolding Numeracy in the Middle Years Research Project 2004-2006

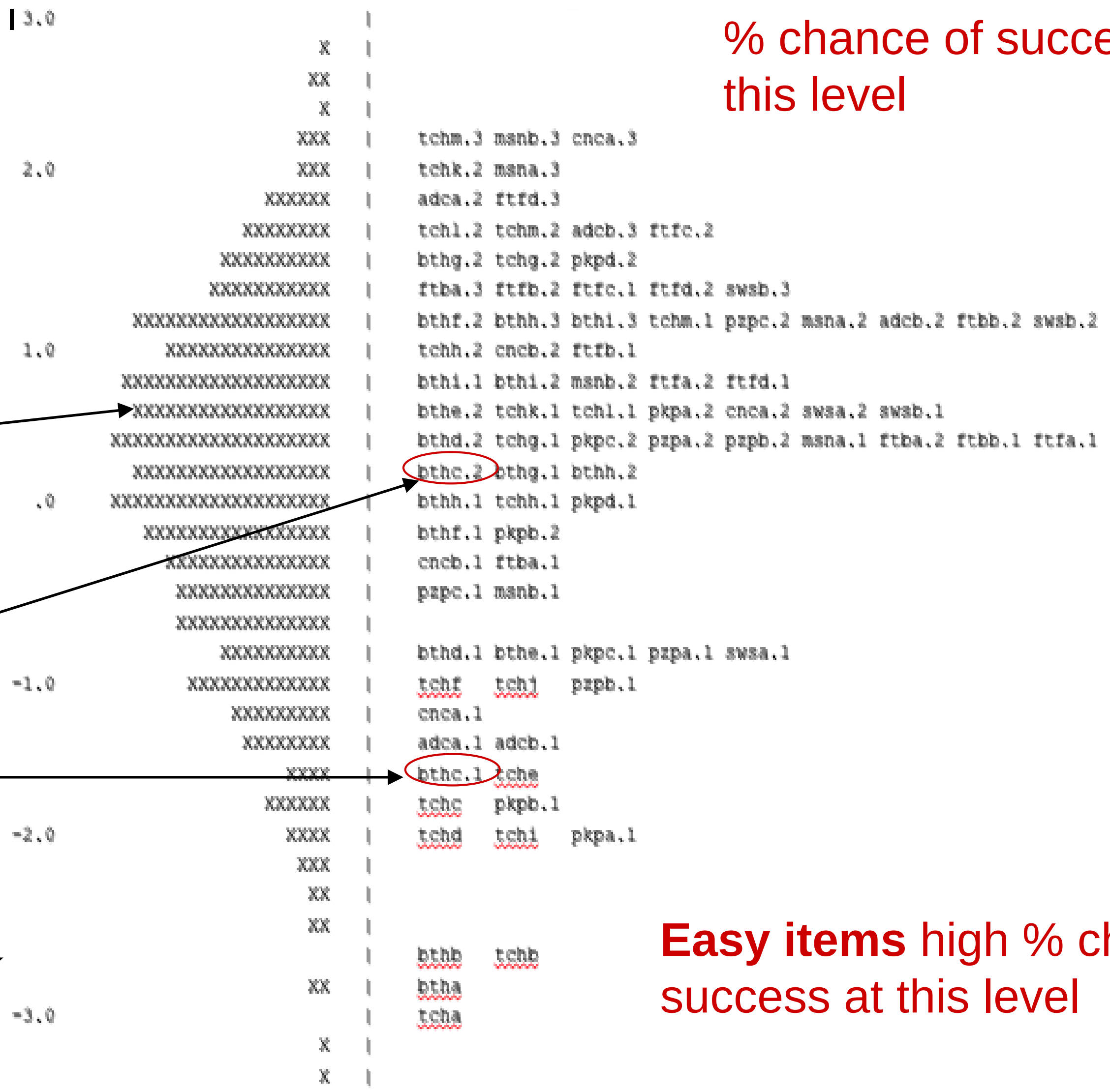
Analysis:

Rasch modelling distributes student scores by item difficulty:

- X's indicate student scores, located on basis of 50% success
- Letter codes indicate items (e.g., bthc indicates *Butterfly House*, part c)
- Numeric code indicates scoring rubric (e.g., score of 1 on bthc task)

Item analysis supported the identification of the ***Learning Assessment Framework for Multiplicative Thinking*** (LAF) comprising eight developmental Zones

Difficult items very low % chance of success at this level

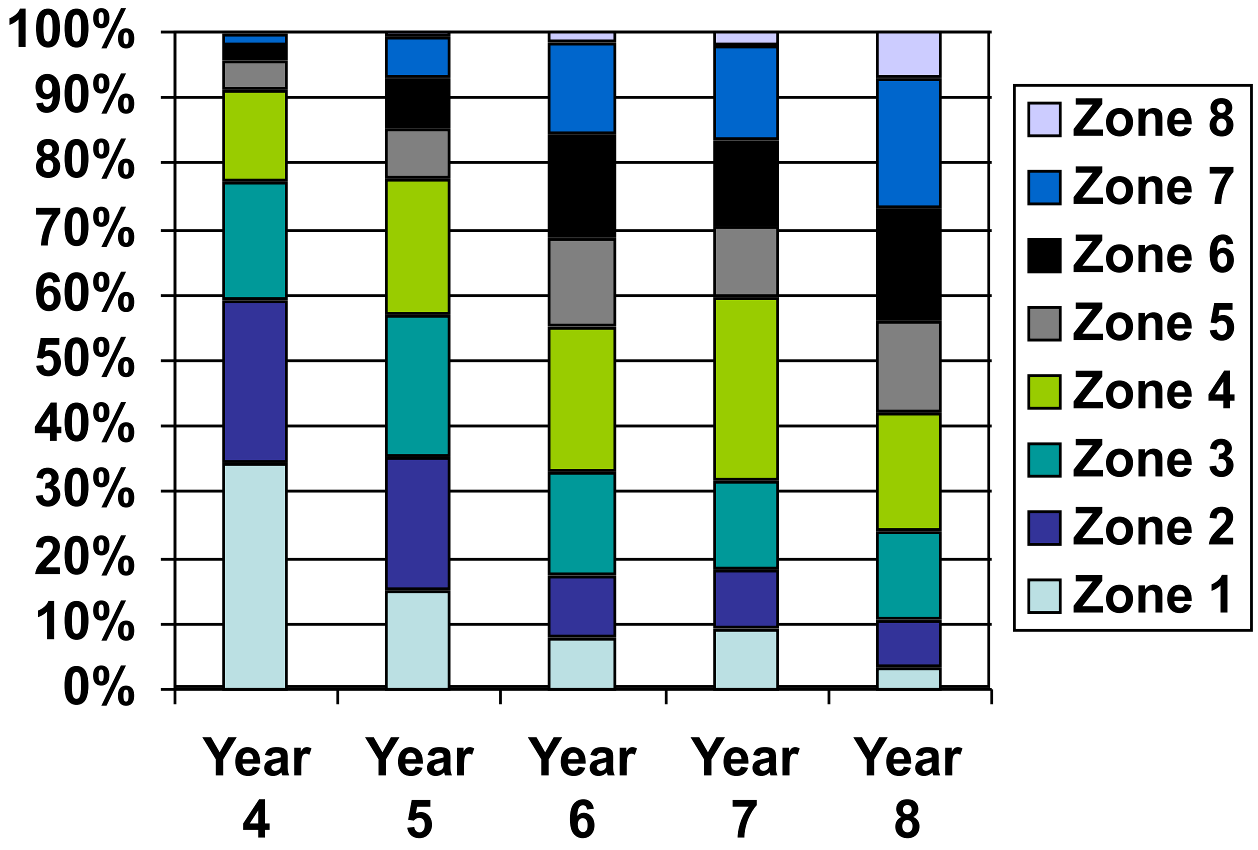


Easy items high % chance of success at this level

Variable map showing item thresholds for multiplicative assessment tasks (N=3167, SNMY, 2004)

Results … Access to Multiplicative thinking confirmed as the reason for the 6-7-year range in each year level.

Zone 4 of the LAF can be viewed as a transitional zone from additive to multiplicative thinking, suggesting that about **40% of Year 7 and 30% of Year 8 students might be deemed to be 'left behind'** in terms of curriculum expectations … (Siemon et al, 2006)



Proportion of Students at each Zone of the LAF by Year Level, Initial Phase of SNMY, May 2004 (N=3169)

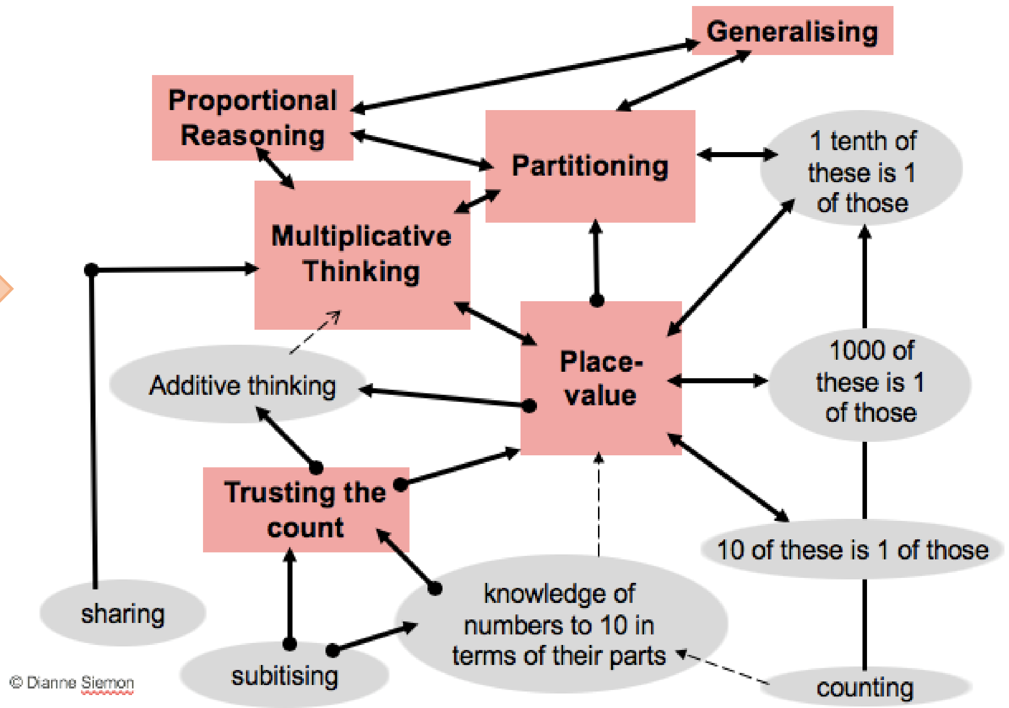
The Big ideas in Number

LAF Zone	Level Description
8	Solves and justifies a wide range of problems involving unfamiliar ~~multiplicative situations~~ including fractions and decimals, solves complex proportional reasoning problems, formally describe patterns in terms of ~~general rules, solves~~ complex, open-ended problems
7	Compare, order, sequence, represent, and rename whole numbers, fractions, decimals, integers, inverse and identity relations, structure of place value system, recognise, describe and apply relationships between variables, algebraic processes, ratio,
6	Extend decimal fractions, use partitio processes for dealing with all four ope notion of variable, formally describe p
5	Uses **partitioning** strategies locate ar value ~~larger whole numbers~~ and tenth multiplying and dividing, area idea, Cartesian product problems, factors and multiples, strategies for adding and subtracting unlike fractions
4	More efficient strategies for multiplying and dividing larger whole numbers, Tenths as a new place-value part, use partitioning strategies to compare fractions, times as many idea
3	**Place-value** based strategies, simple proportion problems, Cartesian ~~product, thirding~~ and fifthing partitioning strategies, key fraction generalisations, works with simple patterns
2	More efficient strategies for counting large collections, array/region-based strategies for multiplication facts, halving partitioning strategies to create fraction representations, key fraction generalisations, extended place value
1	**Trust the count** (part-part-whole knowle ~~addition and subtraction~~, 2 and 3-digit pla arrays and regions

Item analysis pointed to a small number of **BIG IDEAS** in Number that needed to be developed and consolidated over time

The *Learning Assessment Framework for Multiplicative Thinking* (SNMY 2004-2006)

Assessment for Common Misunderstandings F-10
(AfCM, 2006)



What is a ‘big idea’ in this context?

- An idea, strategy, or way of thinking about some key aspect of mathematics, **without which students’ progress in mathematics will be seriously impacted** (e.g., *trusting the count, place value*)
- Encompasses and **connects many other ideas** and strategies (e.g., *multiplicative thinking*)
- Provides an **organising structure** or a frame of reference that supports further learning and generalisations (e.g., *place value, multiplicative thinking, partitioning*)
- Cannot be completely defined but can be **observed in activity** (e.g., *partitioning, proportional reasoning, generalising*).

(Siemon 2006, 2011)

Targeted teaching

Is a form of differentiation that is specifically concerned with addressing students' learning needs in relation to a **small number of really 'big ideas' in Number without which students' progress in school mathematics will be seriously impacted**.

Assessment for Common Misunderstandings (Siemon, 2006)

Goss, P., Hunter, J., Romanes, D., & Parsonage, H. (2015). *Targeted teaching: how better use of data can improve student learning*. Grattan Institute

Targeted Teaching requires:

- **assessment techniques** that expose student thinking in relation to specific aspects of each big idea;
- a **clear understanding of the key mathematical ideas, representations and strategies** that are likely to make a difference, how they are developed over time, and how they are connected;
- **interpretations of what those responses might mean** and **practical suggestions** about where to start teaching

(Siemon, 2006, 2011)

Valid assessment tools

Evidenced-based Learning Progressions

Targeted Teaching Advice

For teachers and school leaders this requires:

- a **commitment to undertake and act on the evidence** to inform both in-the-moment and future teaching to better target the learning needs of all students;
- an **expanded repertoire of teaching approaches** that accommodate and nurture discourse, help uncover and explore student's ideas, and ensure all students can participate in the enterprise;
- **sufficient time** with students to develop trust and supportive relationships; and
- **flexibility** to spend time with those who need it most.

(Siemon & Virgona, 2001)

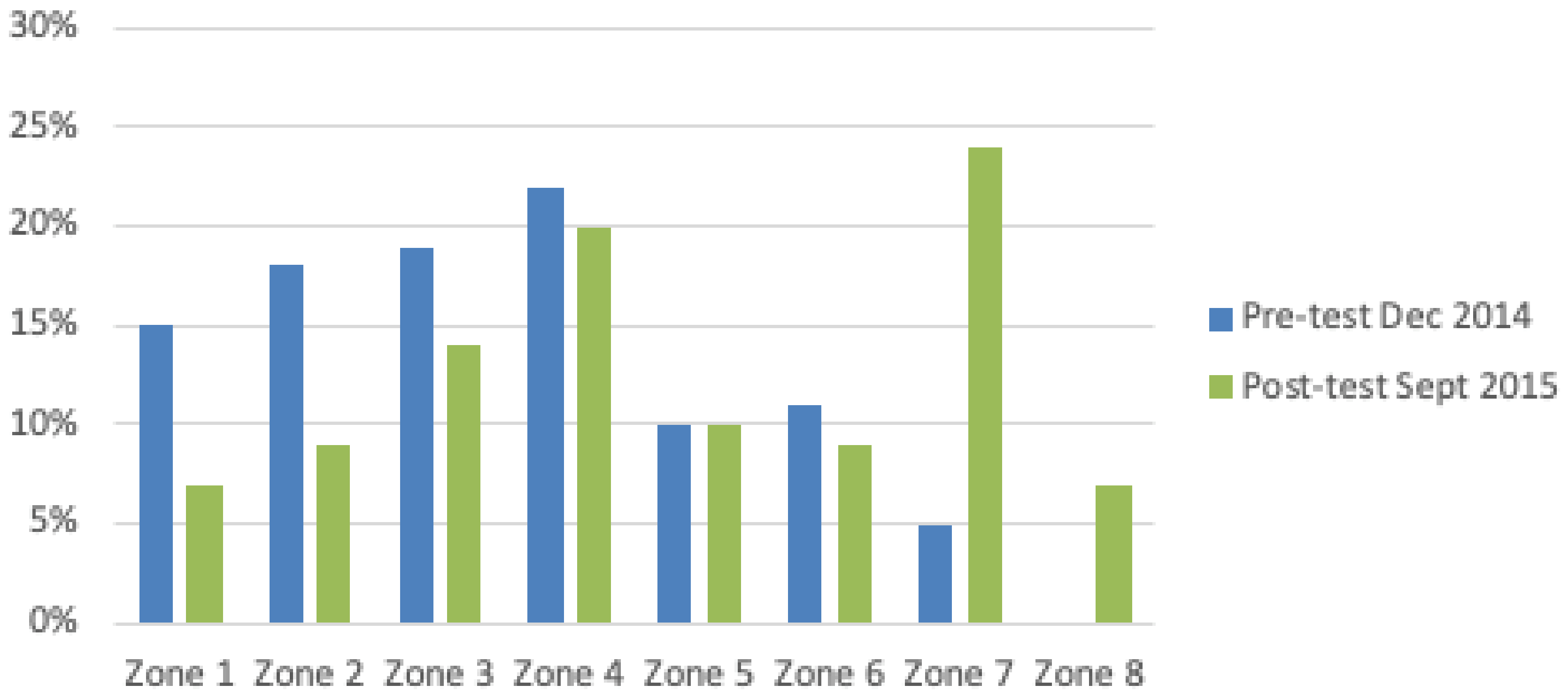
Targeted teaching works

PhD study within the SNMY Project demonstrated major shifts in achievement against the *Learning and Assessment Framework for Multiplicative Thinking* (LAF) as a result of an 18 week, 2 sessions per week teaching program (Breed, 2011).

Participants:	9 Year 6 students identified at Zone 1 of the LAF in May 2004
Results:	8 students achieved at Zone 5 of the LAF in November 2005 (1 top of Zone 4)

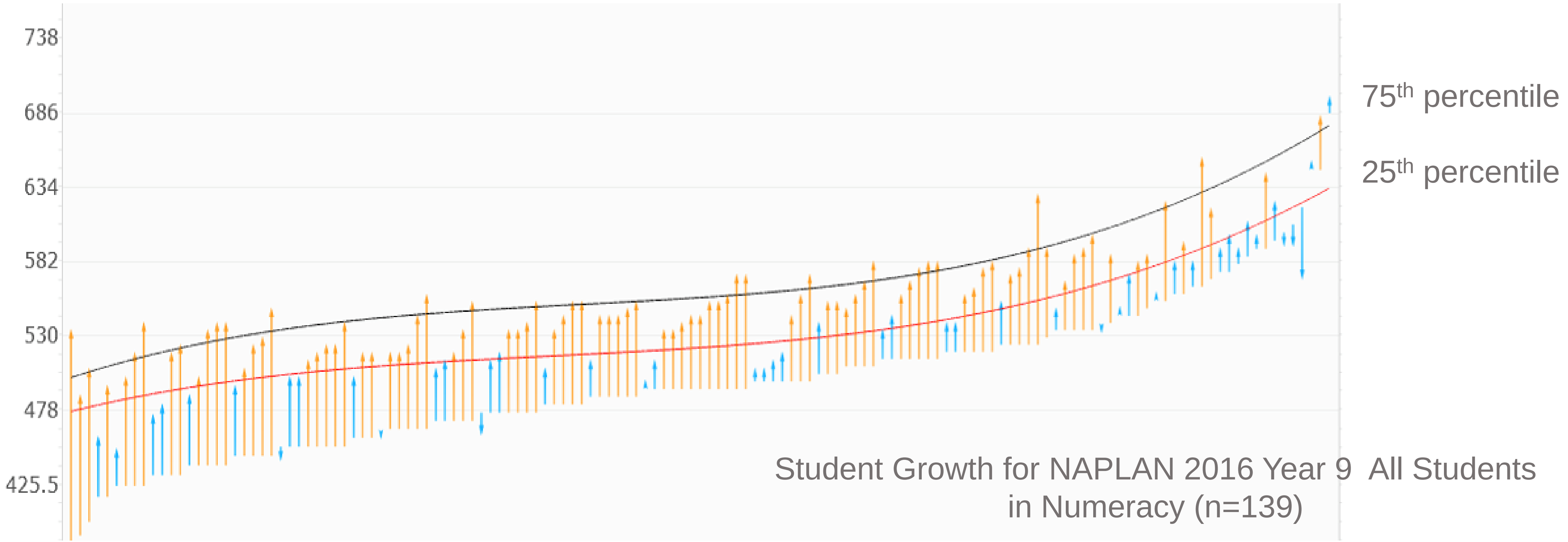
Breed, M. (2011). *Constructing paths to multiplicative thinking*, unpublished PhD Thesis, RMIT University

Results from an RMFII school who used the SNMY resources and the *Learning Assessment Framework for Multiplicative Thinking* to inform a targeted teaching approach in Year 8 in 2015



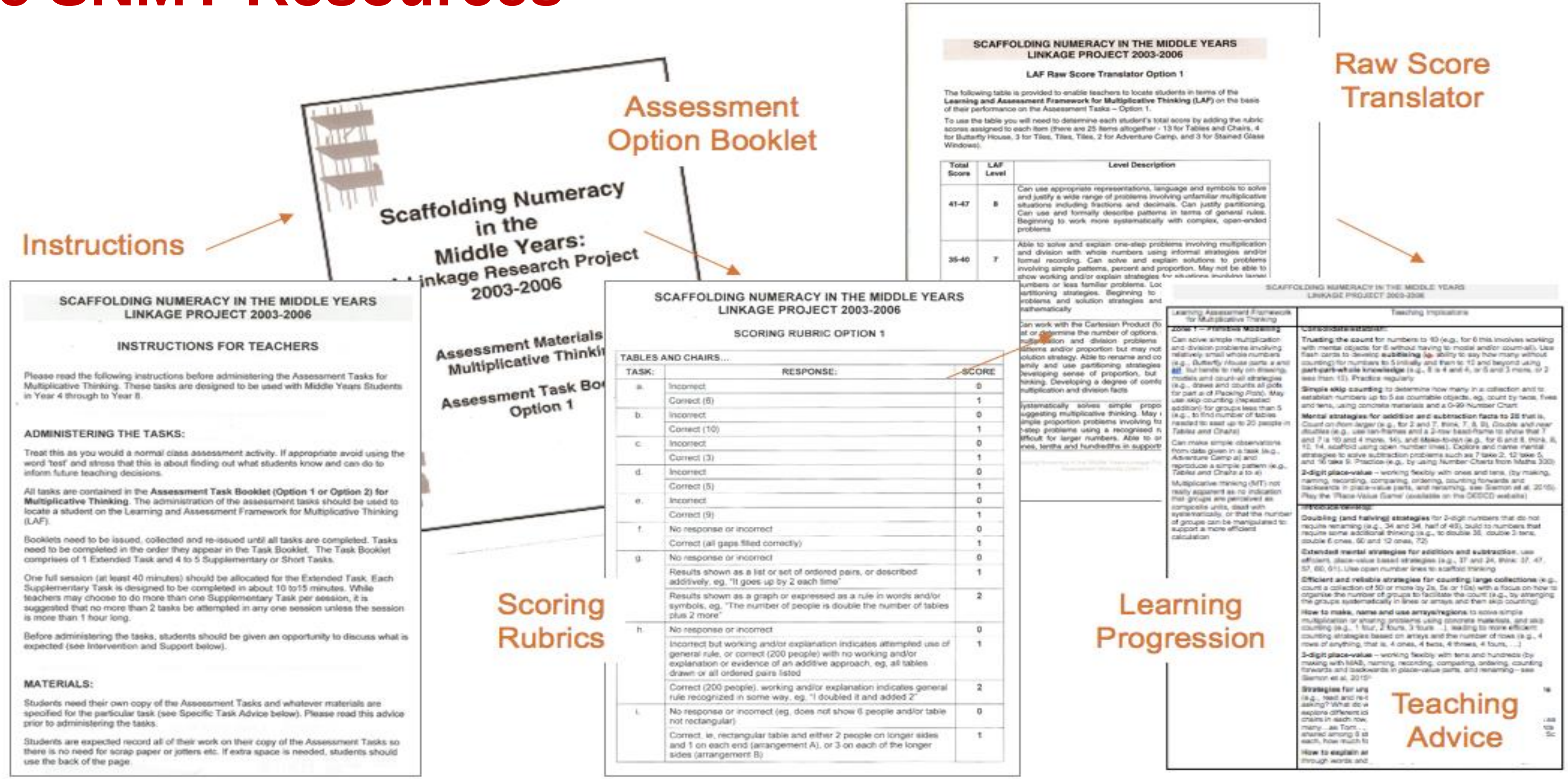
Percentage of students in each Zone of the SNMY LAF in August 2014 (n = 141) and September 2015 (n = 152) (Prasad, November 2016)

The Year 9 NAPLAN results for the same school in 2016 suggest that the growth in learning was sustained and transferable



Growth in student learning was demonstrated for all bar 5 students and 65% of students grew by more than the expected growth - well above State-wide expectations

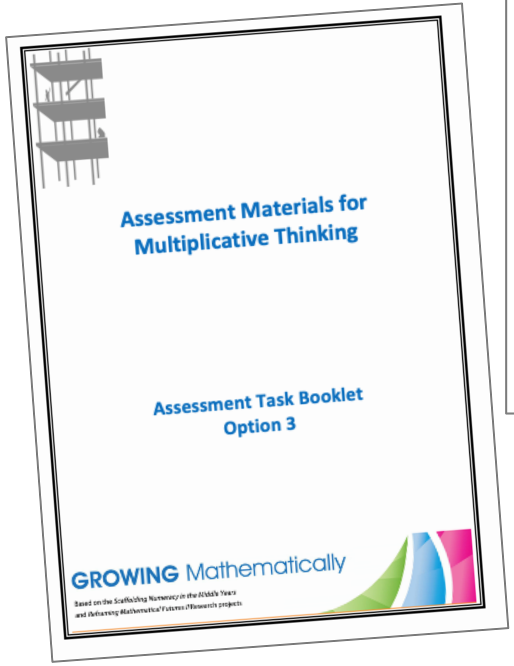
The SNMY Resources

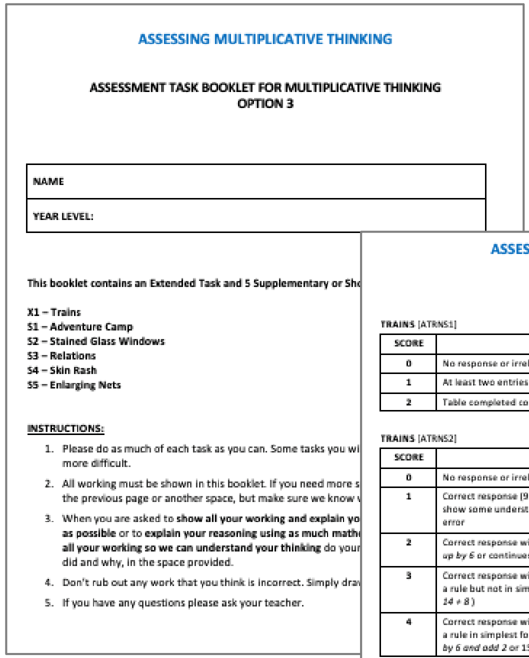


https://www.education.vic.gov.au/school/teachers/teachingresources/discipline/maths/assessment/Pages/scaffoldnum.aspx

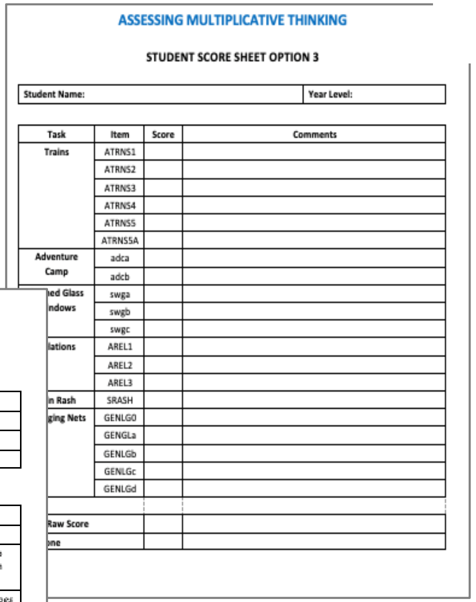
Two new assessment options

Option Booklets

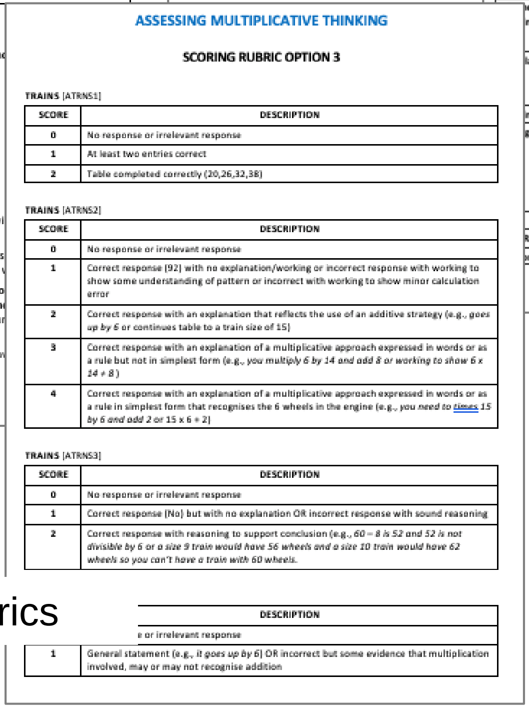




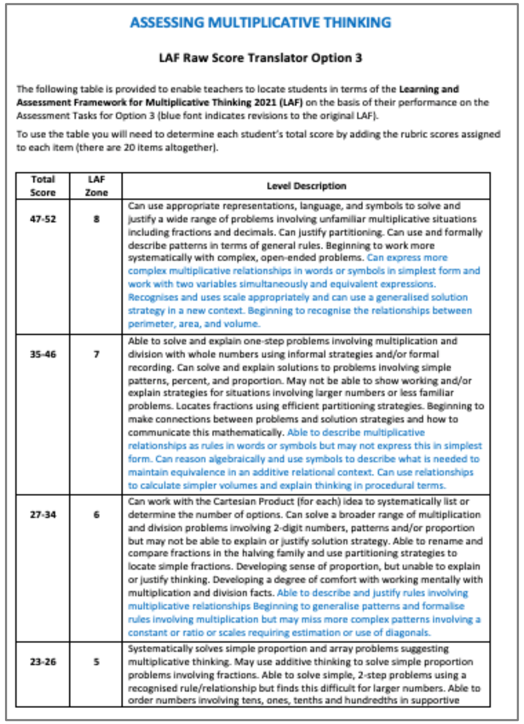
Student Response Sheet



Scoring Rubrics



Raw Score Translator showing revised learning progression for MT



From Growing Mathematically: Multiplicative Thinking. A joint initiative of AAMT and RMFII Project team. Resources available from: http://www.mathseducation.org.au

An updated LAF

8 differentiated levels (Zones) of multiplicative thinking

Teaching advice to consolidate and establish

Teaching advice to introduce and develop

Learning Assessment Framework for Multiplicative Thinking 2021 (SNMY 2004-6, GM-MT 2020)

Zone 3: Sensing	**Consolidate/establish:** **Ideas and strategies** introduced/developed in the previous Zone
Demonstrates intuitive sense of proportion (e.g., partial solution to *Butterfly House f*, correct response to *Lemonade Recipe 1*) and partitioning (e.g., *Missing Numbers b*). Works with 'useful' numbers such as 2 and 5, and strategies such as doubling and halving (e.g., *Packing Pots b*, and *Pizza Party c*). May list all options in a simple Cartesian product situation (e.g., *Canteen Capers b*), but cannot explain or justify solutions. Uses abbreviated methods for counting groups such as doubling and doubling again to find 4 groups of, or repeated halving to compare simple fractions (e.g., *Pizza Party c*). Beginning to work with larger whole numbers but tends to rely on count all methods or additive thinking to solve problems (e.g., *Stained Glass Windows a* and *b*, *Tiles, Tiles, Tiles b*). Can maintain equivalence across the equals sign (e.g., *Relations 1*) and extend patterns but may not be able to explain (e.g., *Trains 2*) or explanation relies on additive thinking (e.g., *Board Room Tables 4*). Beginning to recognise the importance of scale (e.g., *Park Map A1*).	**Introduce/develop:** **Place value based strategies** for informally solving problems involving single-digit by two-digit multiplication (e.g., for 3 twenty-eights, THINK, 3 by 2 tens, 60 and 24 more, 84) mentally or in writing. Initial **recording to support place value** for multiplication facts (see *Siemon et al, 2021* and *There's More to Counting Than Meets the Eye*). More **efficient strategies for solving number problems involving simple proportion** (e.g., recognise as two-step problems, What do I do first? Find value for common amount. What do I do next? Determine multiplier/factor and apply. Why?). How to **rename number of groups** (e.g., think of 6 fours as 5 fours and 1 more four), Practice (e.g., by using 'Multiplication Toss Game'). Re-name composite numbers in terms of equal groups (e.g., 18 is 2 nines, 9 twos, 3 sixes, 6 threes). **Cartesian product** or **for each** idea using concrete materials and relatively simple problems such as 3 tops and 2 bottoms, how many outfits, or how many different types of pizzas given choice of small, large, medium and 4 varieties? Discuss how to recognise problems of this type and how to keep track of the count such as draw all options, make a list or a table (tree diagrams appear to be too difficult at this level, these are included in Zone 5). How to **interpret problem situations and solutions relevant to context** (e.g., Ask, What operation is needed? Why? What does it mean in terms of original question?). Simple, practical division problems that require the **interpretation of remainders** relevant to context. **Practical sharing situations that introduce names for simple fractional parts beyond the halving family** (e.g., thirds for 3 equal parts/shares, sixths for 6 equal parts etc) and help **build a sense of fractional parts** (e.g., 3 sixths is the same as a half or 50%, 7 eighths is nearly 1, "2 and 1 tenth" is close to 2). Use a range of continuous and discrete fraction models including mixed fraction models. **Thirding and fifthing partitioning strategies** (see Siemon et al, 2021) through paper folding (kinder squares and streamers), cutting plasticine 'cakes' and 'pizzas', sharing collections equally (counters, cards etc), **apply thinking involved to help children create their own fraction diagrams (regions) and number line representations** (see Siemon et al 2021). Focus on making and naming parts in the thirding and fifthing families (e.g., 5 parts, fifths) including mixed fractions (e.g., "2 and 5 ninths") and informal recording (e.g., 4 fifths), no symbols. Revisit key fraction generalisations (see Level 2), include whole to part models (e.g., partition to show 3 quarters) and part to whole (e.g., if this is 1 third, show me the whole) and use diagrams and representations to **rename related fractions.** **Extend partitioning strategies** to construct number line representations. Use multiple fraction representations. **Key fraction generalisations** – equal parts, as the number of parts increase the size of the part gets smaller; the number of parts names the part (e.g., 8 parts, eighths) and the size of the part depends upon the size of the whole. **Recognise and work with more complex number patterns** – explore relational thinking situations when using different operations and use missing numbers on both sides to generate generalisations. (e.g., *Maths300: Arithmagons, nRich: Super Shapes*), model and discuss how to describe, explain, and generalise simple patterns and relations. **Use simple scales** (e.g., 1 grid square length to 10 units)

The *Revised Learning and Assessment Framework for Multiplicative Thinking* (2021)

Implementing a Targeted Teaching Approach

There is no one way to do this - some suggestions:

- Group students across two or more classes in Zone groupings for 1 maths lesson a week, with support from additional staff if possible
- Use a double period once a fortnight to group two or more classes in Zone groupings as above
- Implement an intervention program for ‘at risk’ students (e.g., staff member works with a small group of three or so students for 20- 25 minutes preferably within the classroom)
- 10-week focus on tasks that afford opportunities to engage with key ideas in mixed ability groups

See Case Studies from Growing Mathematically: Multiplicative Thinking website: http://www.mathseducation.org.au

An important note:

What is Introduced and Developed at one Zone is Consolidated and Established at the next Zone

Zone 1	C&E	I&D							
Zone 2		C&E	I&D						
Zone 3			C&E	I&D					
Zone 4				C&E	I&D				
Zone 5					C&E	I&D			
Zone 6						C&E	I&D		
Zone 7							C&E	I&D	
Zone 8								C&E	I&D

Zone 2: Intuitive Modelling	
Consolidate & Establish	**Introduce & Develop**
• Doubling (and halving) strategies • Extended mental strategies for addition and subtraction • Efficient and reliable strategies for counting large collections • How to make, name and use arrays/regions • 3 digit place-value • Strategies for unpacking and comprehending problem situations • How to explain and justify	• More efficient strategies for counting groups • Array/region-based mental strategies for multiplication facts to 100 • Commutativity • Informal division strategies • Extended mental strategies for multiplication • Simple proportion problems • How to recognise and describe simple relationships • Language of fractions • Halving partitioning strategy • Key fraction generalisations

Zone 3: Sensing	
Consolidate & Establish	**Introduce & Develop**
• More efficient strategies for counting groups • Array/region-based mental strategies for multiplication facts • Commutativity • Informal division strategies • Extended mental strategies for multiplication • Simple proportion problems • How to recognise and describe simple relationships • Language of fractions • Halving partitioning strategy • Key fraction generalisations (equal parts)	• Place-value based strategies • Simple proportion problems (intuitive) • Cartesian product or for each idea • How to interpret problem situations and solutions relevant to context • Interpretation of remainders • Practical sharing situations to build a sense of fractional parts, • Thirding and fifthing partitioning strategies to create fraction models • Extend partitioning strategies • Key fraction generalisations (number of parts in relation to size)

‘Threads’ can be used to inform task selection

Consolidating & Establishing Zone 1	Introducing & Developing Zone 1/Consolidating & Establishing Zone 2	Introducing & Developing Zone 2
• Trusting the count (subitising and part-part-whole knowledge) • Simple skip counting (2s & 5s) • Mental strategies for addition and subtraction facts to 20 • 2 digit place-value	• Doubling (and halving) strategies • Extended mental strategies for addition and subtraction • Efficient and reliable strategies for counting large collections • How to make, name and use arrays/regions • 3 digit place-value • Strategies for unpacking and comprehending problem situations • How to explain and justify	• More efficient strategies for counting groups • Array/region-based mental strategies for multiplication facts to 100 • Commutativity • Informal division strategies • Extended mental strategies for multiplication • Simple proportion problems • How to recognise and describe simple relationships • Language of fractions • Halving partitioning strategy • Key fraction generalisations

The Assessment for Common Misunderstandings (Big Ideas) Resources

5.3 Understanding Scale Factors Tool[1]

Materials:
2 Pentagon Cards (cut out so that they can be manipulated, see Level 5 Resources)
Dot Paper Worksheet (see Level 5 Resources)
Map Worksheet (see Level 5 Resources) and pen
A ruler

Instructions:
Place the two cards in front of the student and say, **"What can you tell me about these two shapes?** ... Note student's response, then say, **"How would you tell a friend to draw the large shape if you could only show them the small shape?"** Note student's response.

Place the Dot Paper Worksheet in front of the student and say, **"Could you make this shape half as big please?"** Note and retain student's response.

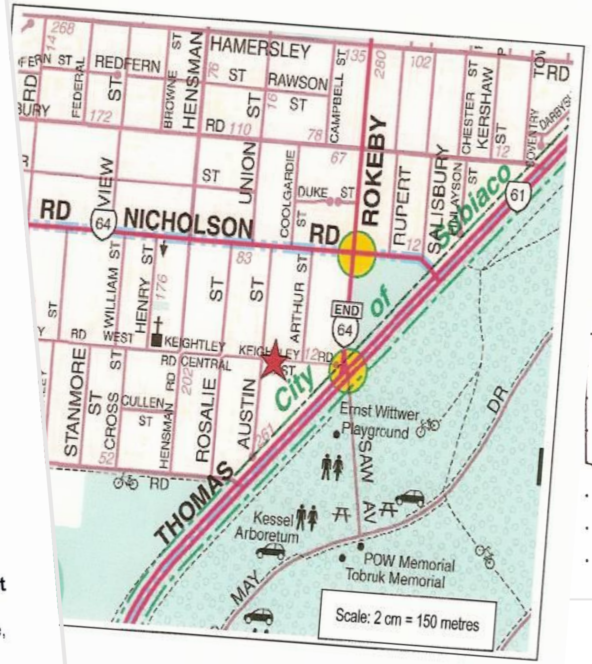
Place the Map Worksheet in front of student and say, **"This is a map of a suburb in Perth. Can you find Nicholson Road?"**.... Point to the scale and ask, **"Can you tell me what this means? ... Can you give me an example?"** ...

If no response, say, **"If you walked the full length of Arthur Street** (indicate this, it is just to the right of the red star), **about how far would you have walked?"** ... Note student's response, then say, **"Jo walks to Rosalie School which is here** (indicate the red star). **She lives on the corner of Nicholson and Rupert Street** (indicate). **About how far does she walk to school in the morning?"** ... Indicate that the bottom of the page can be used for any working required. Note student's response and ask him/her to explain their reasoning.

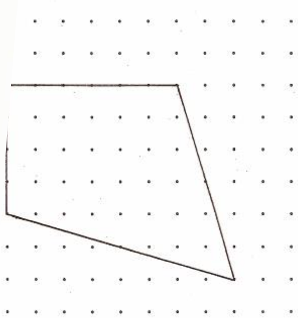
If correct (ie, about 375 metres), say, **"Thuan lives on the corner of Redfern Street and View Street** (indicate upper left hand corner of the map). **About how far does he have to ride to school?"** .... Note student's strategy and response in each case, retain any working.

Diagnostic Interview

Map Worksheet:



Resources



5.3 Understanding Scale Factors Tool

Proportional reasoning is often more apparent in relation to visual images, eg, recognising shapes that have been enlarged or reduced, than it is in word problems that require interpretation relative to context. However, where students have had a limited exposure to the skills and strategies needed to enlarge or reduce shapes and/or to construct and interpret scale drawings there is a distinct possibility that misunderstandings will arise. One of these is the tendency to focus on area when attempting to identify 'how many times larger' one shape is of a smaller, similar shape.

This task examines the extent to which students are able to recognise and describe enlargements, and use a scale factor to reduce a shape and estimate distances on a scale map.

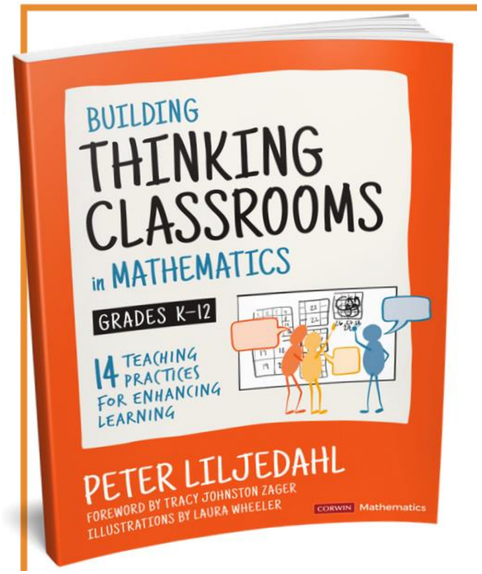
Observed response	Interpretation/Suggested teaching response
Little/no response, possibly recognises shapes and/or notes one is bigger than the other, may be able to explain meaning of map scale	*May not understand the spatial task or have access to the skills and strategies needed to interpret maps* • Ensure that what is meant by statements such as, "3 times as big as" or "half the size of" are understood, ie, they can be modelled and interpreted relative to context • Use peg boards, dot paper, cm grid paper etc to enlarge and reduce shapes by simple scalar amounts, discuss this in terms of what happens to corresponding sides (they are multiplied or divided by the same factor) • Discuss which attribute is relevant and why for 2 D shapes (ie, length not area, as object is to produce similar shapes) • Provide opportunities to work with maps and scale diagrams, make thinking explicit, scaffold appropriate strategies for calculating or estimating distances
Recognises shapes are the same, may identify scale factor (3) but unable to halve the quadrilateral, although may make a start (eg, draw relevant diagonal), explains meaning of map scale but may not be able to use this to provide an example or reasonable estimates for both map questions (eg, may treat as 1 cm is 350 metres)	*Suggests a limited understanding of the scale factor idea for multiplication,* • Use cm grid/dot paper, peg boards etc to review the processes and language involved in enlarging and reducing 2D shapes by a range of different factors starting with simple shapes such as rectangles and moving to more complex shapes such as scalene triangles and irregular quadrilaterals • Practice map reading skills and strategies, talk about the use of scales, construct scale drawings of the classroom, school grounds, students homes and/or backyards etc. discuss equivalent scales (eg, 2 cm to 150 metres is the same as 1 cm to 75 metres) • Explicitly link the use of scales to multiplication using the term *scale factor*, explore the impact of different scale factors, including scale factors less than 1
Recognises scale factor for pentagons, able to halve the quadrilateral, may use diagonal from right angle vertex to opposite vertex to locate corresponding point, can explain meaning of map scale and provide reasonable estimates of distances	*Indicates a solid understanding scale factors and how it relates to multiplication in this context* • Provide opportunities for students to work with an extended range of scale factors, eg, very large whole numbers, decimals, mixed fractions, percentages, ratios, etc • Extend scale drawing skills and strategies to include the idea of perspective and the use of a centre of enlargement (or dilation) • Link solution strategies to proportional reasoning problems more generally, eg, finding for 1 and multiplying or finding for a common composite unit and multiplying • Practice map reading skills and strategies, talk about the use of scales

Teaching Advice

https://www.education.vic.gov.au/school/teachers/teachingresources/discipline/maths/assessment/Pages/misunderstandings.aspx

Targeted Teaching $\neq$ Ability Grouping (Streaming)

- Students need to work in mixed ability groups on rich, accessible, but challenging tasks for the majority of maths time (Boaler, 2008; Sullivan, 2011)
- Flexibility and student choice are key



re(Solve) MATHS BY INQUIRY

maths 300

engaging lessons and professional support

NRICH

https://www.education.vic.gov.au/Documents/school/teachers/teachingresources/discipline/maths/ability-grouping.pdf