Reasoning: “The glue that holds everything together”

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Bringing Maths to Life

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Working Mathematically process: Reasoning

- ★ Understand reasoning as a process
- ★ Teach reasoning effectively
- ★ Assess reasoning consistently

Rationale

The **Rationale** draws on research regarding the positive impact of mathematics education when students enjoy learning Mathematics:

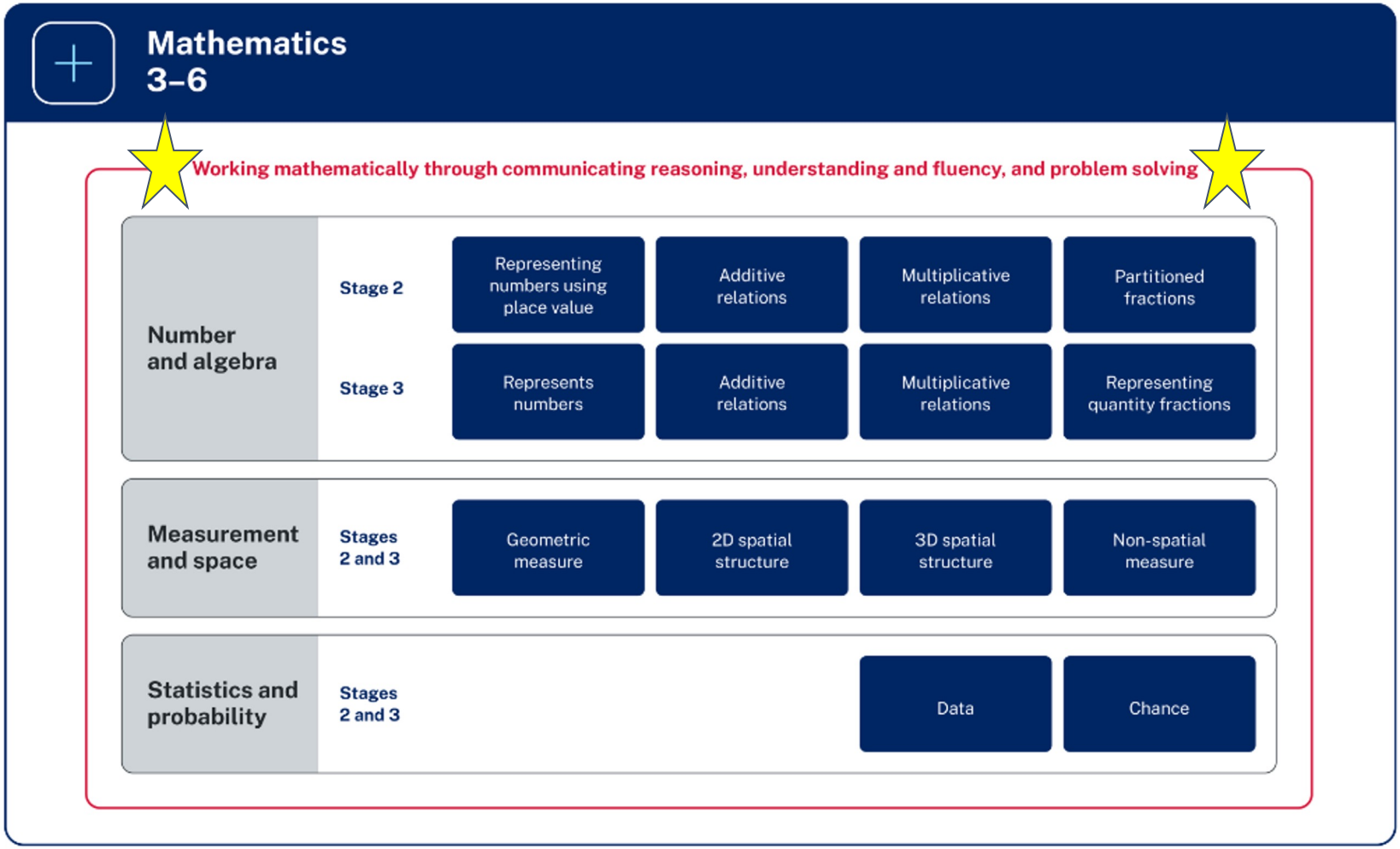
> When students enjoy learning Mathematics, they develop a positive self-concept and become self-motivated learners through active participation in appropriately challenging tasks. This can enhance their resilience in solving mathematical problems relevant to further education *and* their everyday lives.

Aim

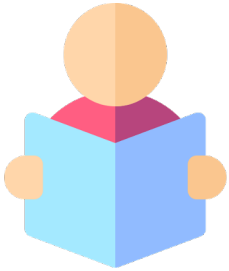
“The aim of Mathematics K–10 is to enable students to become confident users of mathematics, learning and applying the language of mathematics to **communicate efficiently and effectively**.

They develop an increasingly sophisticated **understanding** of mathematical concepts and a **fluency** with mathematical processes that helps them to interpret and **solve problems**.

Students **make connections** within mathematics and connect mathematical concepts with the world around them. They learn to understand and appreciate how *mathematics is a relevant part of their lives*.”


Mathematics
3–6
Working mathematically through communicating reasoning, understanding and fluency, and problem solving
Number and algebra
Stage 2
Representing numbers using place value
Additive relations
Multiplicative relations
Partitioned fractions
Stage 3
Represents numbers
Additive relations
Multiplicative relations
Representing quantity fractions
Measurement and space
Stages 2 and 3
Geometric measure
2D spatial structure
3D spatial structure
Non-spatial measure
Statistics and probability
Stages 2 and 3
Data
Chance

Professional Perusal!



Scan NSW Curriculum document: ***Reasoning in Mathematics K–6***

https://tinyurl.com/NSWreasoning

★ How is ‘reasoning’ different to ‘explaining’?

★ What types of reasoning does the syllabus require students to learn?

“Reasoning is the glue that holds everything together,

the lodestar that guides learning” (Kilpatrick et al., 2001, p.129).

Reasoning: Match Masters!

https://tinyurl.com/reasonmatch

PROBLEM 9

e.g. Stage 2A Representing Number

Compare and describe the relative size of numbers by positioning numbers on a number line (Reasons about quantity)

PROBLEM 5

e.g. Stage 2B Multiplicative Relations

Complete number sentences involving multiplication & division by calculating miss… …out relat…

PROBLEM 1

e.g. Stage 2A Multiplicative Relations

Create and continue a variety of number patterns that increase or decrease by a constant amount (Reasons about patterns)

PROBLEM 2

e.g. Stage 3B Multiplicative Relations

Determine a rule describing the relationship between the bottom number and the top number in a table (Algebraic reasoning)

PRO…

e.g. …ions

Recr… …fractional part …

PROBLEM 4

e.g. Stage 2A Data

Represent the same dataset using more than one type of display and compare the displays (Statistical reasoning)

PROBLEM 6

e.g. Stage 3A 2D Spatial Structure

Recognise that triangles and quadrilaterals can be classified in more than one way (Reasons about spatial relations)

PROBLEM 7

e.g. Stage 2B 3D Multiplicative Relations

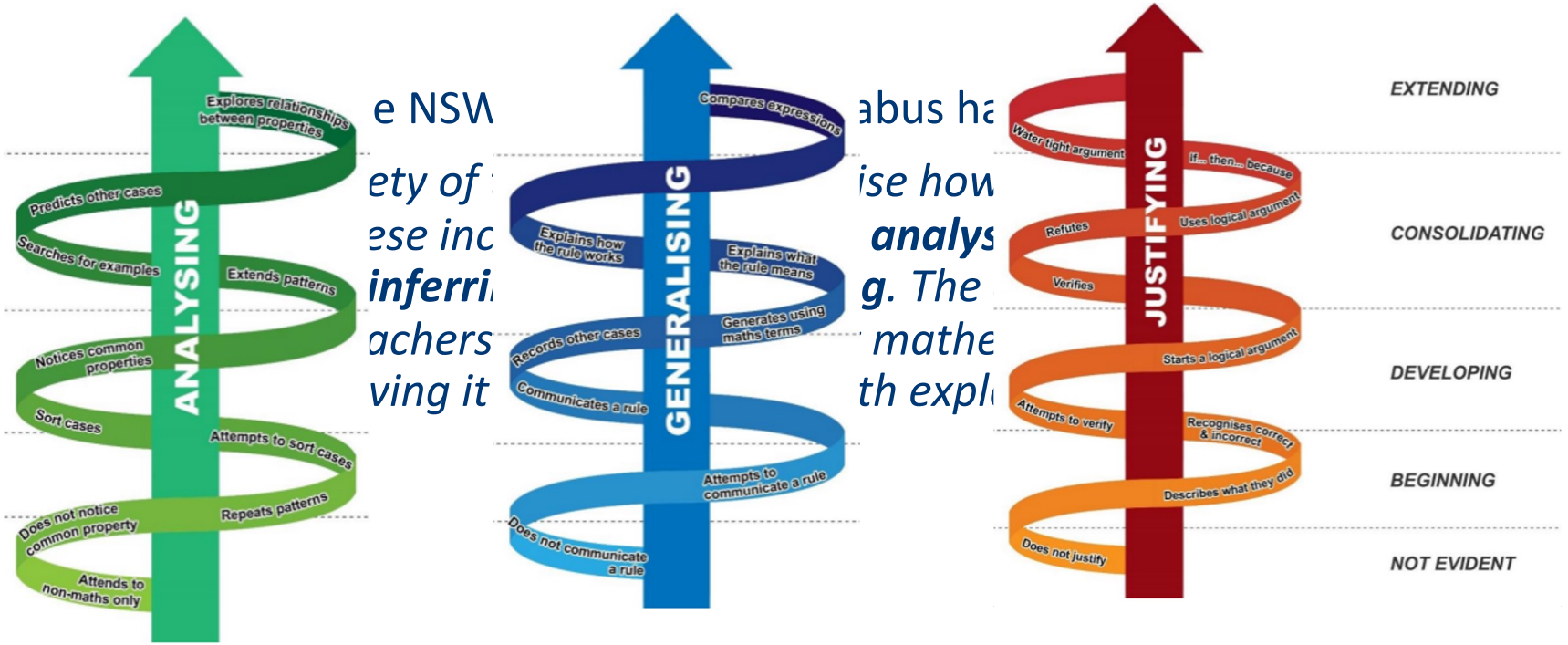
Use the associative property within multiplication to regroup the factors (Reasons about spatial structure)

PROBLEM 3

e.g. Stage 3A 3D Spatial Structure

Visualise and sketch three-dimensional objects from different views, including top, front and side views (Reasons about spatial orientation)

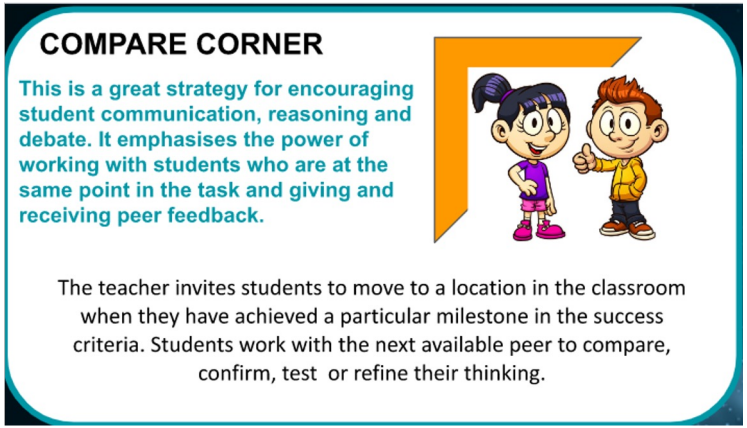
Teaching reasoning: Three key actions

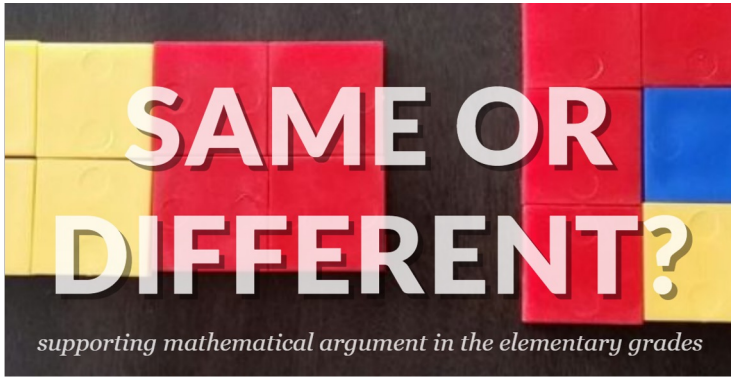


reSolve website funded by the Australian Government Department of Education and managed by the Australian Academy of Science

Questions to encourage analysing

- What is the same and different about …?
- What stays the same and what changes?
- Sort or organise the following according to …
- Alter an aspect of something to see an effect. If we change this what will happen?
- What follows from this? What do you think will happen next if we do this?
- What do you notice…?
- When is it true?
- Is it just sometimes true, or is it always true?





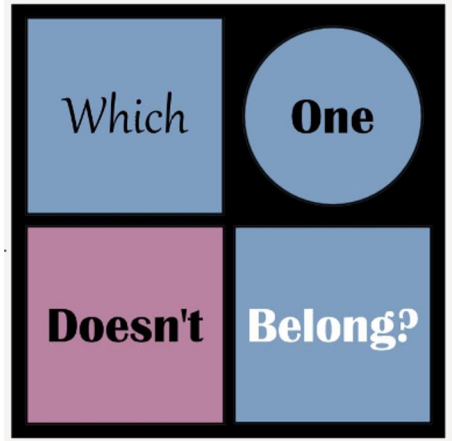
Questions to encourage generalising

- How can you describe what is the same?
- What is the rule?
- What is the pattern here?
- How can you describe the pattern?
- What happens in general?
- Is that … (pattern) always going to work?
- Are there other examples that fit the rule?
- How could you explain the rule to someone else?

THINK ALOUD

An ideal strategy for increasing student reasoning and communication and making visible where student thinking is efficient, needs to be be refined or breaks down

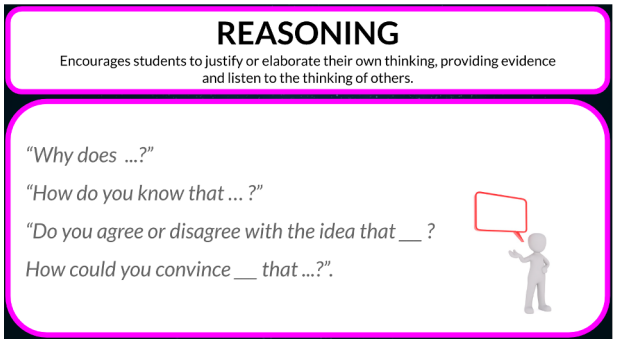
A think aloud can be led by a teacher or a student. Students can work in pairs sharing and recording think alouds in the classroom. Think alouds can be captured in writing or using technology and provide a powerful form of assessment.



Questions to encourage justifying

JUSTIFYING

- Is this conjecture just sometimes true, or always true?
- How do you know?
- How could we show or prove that it is true?
- True or false? Why? Let's justify.
- Convince me...
- How can we be sure...?
- Tell me what is wrong with....
- Explain - why does this (process/rule/result) work?
- Can you go through that step by step?
- Can you explain that step by step?
- Why?
- If...then...



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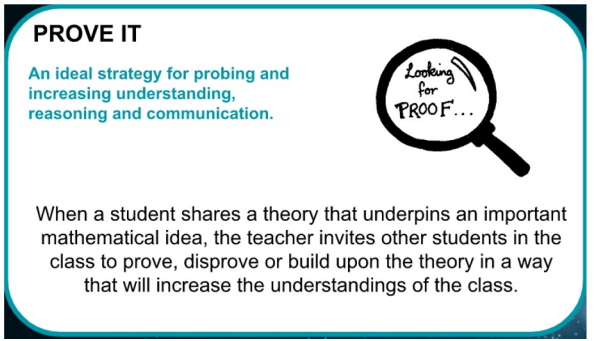


Assessing reasoning: Rubrics

	ANALYSING	GENERALISING	JUSTIFYING
NOT EVIDENT	• **Does not notice** common property or pattern.	• **Does not communicate** a common property or rule (conjecture) for a pattern.	• **Does not justify**.
BEGINNING	• **Recalls** random known facts or **attempts to sort** examples or **repeats** patterns.	• **Attempts to communicate** a common property or rule (conjecture) for a pattern.	• **Describes** what they did and **recognises** what is correct or incorrect. • **Argument** is not coherent or does not include all steps.
DEVELOPING	• **Notices** a common property, or **sorts and orders** cases, or **repeats and extends** patterns. • **Describes** the property or pattern.	• **Generalises: communicates** a rule (conjecture) using mathematical terms, and records other cases or examples.	• **Attempts to verify** by testing cases, and **detects and corrects** errors or inconsistencies. • Starting statements in a **logical argument** are correct.
CONSOLIDATING	• **Systematically searches** for examples, extends patterns, or analyses structures, to form a conjecture. • Makes **predictions** about other cases.	• **Generalises: communicates** a rule (conjecture) using mathematical symbols and **explains what** the rule means or **explains how** the rule works using examples.	• **Verifies** truth of statements by confirming all cases or **refutes** a claim by using a counter example. • Uses a correct **logical argument.**
EXTENDING	• **Notices and explores** relationships between properties.	• **Generalises** cases, patterns or properties using mathematical symbols and **applies** the rule. • **Compares** different expressions for the same pattern or property to show equivalence.	• Uses a **watertight logical argument**. • **Verifies** that the generalisation holds for *all* cases using logical argument.
Comments (feedback, reasoning prompts for further development):			

Video of student explaining reasoning

With thanks to St Patrick's Mortlake

Assessing reasoning

Sample assessment

A Magic V is a number puzzle. The aim is to arrange five consecutive numbers so that the sum of each 'arm' of the V is the same. Below are two Vs. The left V is a Magic V (4+2+3=9 and 5+1+3=9); the V on the right is not. Each number can be used only once.

Sam said, "It is impossible to make a Magic V with an even number at the bottom with the set of numbers 1 to 5."

Is Sam right? Explain why or why not.

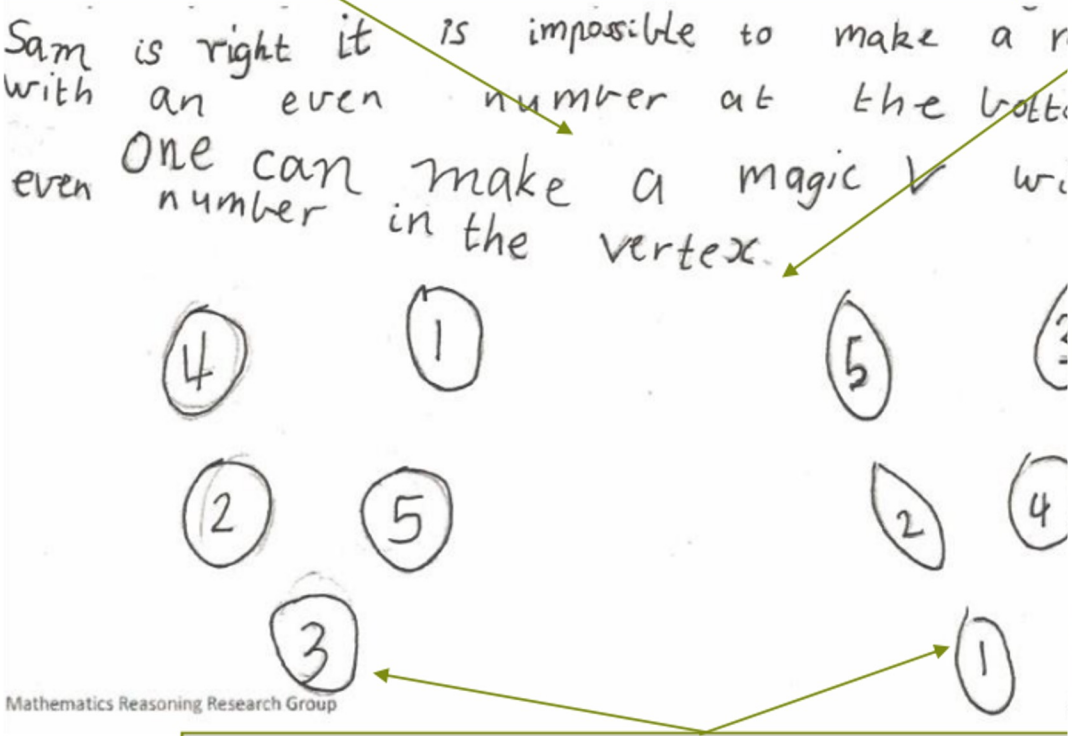
ENABLING PROMPT: Make as many Magic Vs as you can with 5 at the vertex. Write the totals of the numbers on the arms.

EXTENDING PROMPTS: Can it be done with numbers 2-6? Generalise Sam's conjecture to all sets of five consecutive numbers.

What is the same and different about ...?	Alter an aspect of something to see an effect. If we change this what will happen?	What is the pattern here?	Is that... (pattern) always going to work?	Convince me...	Explain - why does this work?

JUSTIFYING: Recognises what is correct or incorrect using materials, objects or words.

However, the second part of the statement does not make sense. The teacher would need to speak to the student to clarify what they mean. (Maybe just a slip?)



ANALYSING: Notices similarities across examples.

The student notices the importance of the vertex in creating a Magic

Student Name: Work Sample 2 Reasoning Task: MAGIC V Date:

Observation of student's reasoning:

* Adding total of arms & comparing 'Vs' with an odd/even number at vertex.
* developing a logical argument but no 'why'

	Analysing	Generalising	Justifying
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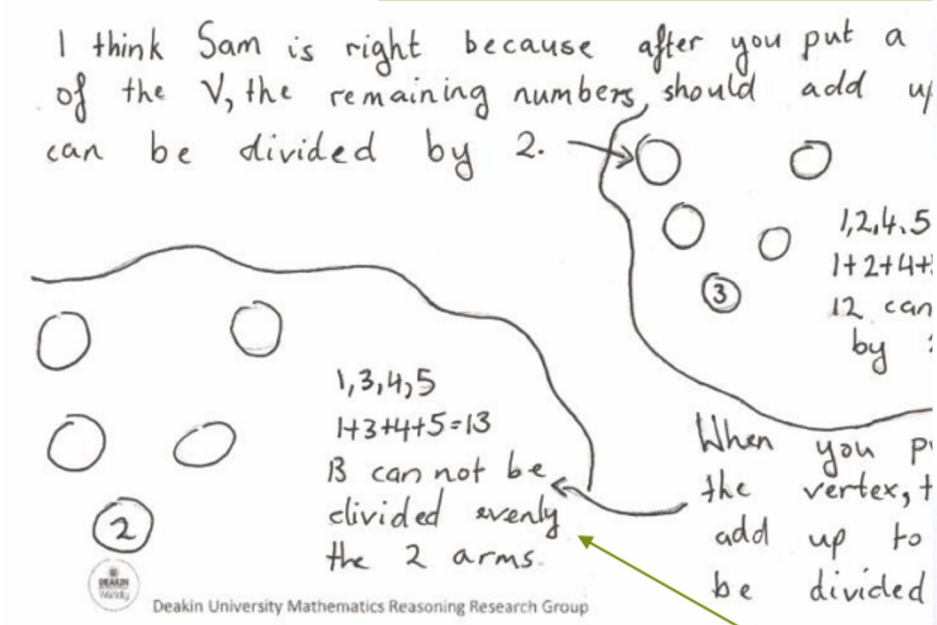
Beginning	• **Recalls** random known facts or **attempts to sort** examples or **repeats** patterns.	• **Attempts to communicate** a common property or rule for the pattern.	• **Describes** what they did and **recognises** what is correct or incorrect. • **Argument** is not coherent or does not include all steps.
Developing	• **Notices** a common property, or **sorts and orders** cases, or **repeats and extends** patterns. • **Describes** the property or pattern.	• **Generalises: communicates** a rule (conjecture) using mathematical terms and records other cases or examples.	• **Attempts to verify** by testing cases and **detects and corrects** errors or inconsistencies. • Starting statements in a **logical argument** are correct.
Consolidating	• **Systematically searches** for examples, extends pattern or analyses structure to form a conjecture. • Makes **predictions** about other cases. * 'what will...'	• **Generalises: communicates** a rule using mathematical symbols and **explains what** the rule means or **explains how** the rule works using examples.	• **Verifies** truth of statements by confirming all cases or **refutes** a claim by using a counter example. • Uses a correct **logical argument.**
Extending	• **Notices and explores** relationships between properties.	• **Generalises** cases, patterns or properties using mathematical symbols (including algebraic symbols) and **applies** the rule. • **Compares** different expressions for the same pattern or property to show equivalence.	• Uses a **watertight logical argument**. • **Verifies** that the generalisation holds for *all* cases using logical argument.

Comments (feedback, reasoning prompts for further development):

* Look closer at properties of odd/even numbers
* Develop 'justification' → look at reasoning prompts

JUSTIFYING: Verifies the statement is true o

The next step is to develop a watertight argume



ANALYSING: Extending

GENERALISING: Extending

JUSTIFYING: Extending

Teacher Prompt:

What if we use numbers from 2-6?

What if we have more even than odd numbers? Can you convince us that an even number at the bottom will make a Magic V? Why?

ANALYSIN
relationsh
of pattern

The stude
odd and e
the signifi
in total.

Student Name: Work Sample 4 Reasoning Task: Magic V Date:

Observation of student's reasoning:

* Watertight argument about odd/even & totals of arms - gave examples to support justification

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Comments (feedback, reasoning prompts for further development):

* Enhance justification by exploring other possible Magic Vs e.g using numbers 2-6.

Moving forward with Working Mathematically

- **Understanding** reasoning
- **Teaching** reasoning
- **Assessing** reasoning

*What is **one thing** you will do in your mathematics lessons tomorrow to include working mathematically processes?*

Andrea de Carvalho

[Australian Curriculum Working Mathematically Resources]